A Brief Introduction to Post-Quantum Cryptography

Fernando Virdia

NOVA.ID.FCT
Universidade NOVA de Lisboa

Outline

- Motivation: Cryptography and Quantum Computing
- Foundations: New Hardness Assumptions
- Standards: The US NIST process
- Deployment: Some of the challenges

Slides @ https://fundamental.domains

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Together, they enable the large-scale deployments of cryptography that we see today on the Internet, and in payment systems.

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- We formalise these problems into concise "hardness assumptions".
- Part of the job of cryptographers is identifying hardness assumptions, trying to break them, and constructing primitives from them.

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These problems received a lot of study, and are used everywhere in software and hardware.

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NOTE: We cannot have absolute certainty that the problem is hard. (Eg., maybe $P = NP$)

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- But there exist much faster attacks, such as the (general number field sieve, GNFS) that takes

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Do we know for sure no better attack exists? No! The only option is to make our best effort to study the problem and new possible attacks.

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Quantum computers would represent a new kind of "resource" in the hands of attackers.

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- (You can also apply changes to α and β.)
- You then read ("measure") the register.

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Unfortunately, yes. So far, in the form of two algorithms: Grover's and Shor's.

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- Grover's algorithm lets you find x in $O(\sqrt{N})$ superposed comparisons.

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Every cipher is automatically weaker! Need keys twice as long!

(See my talk on Friday about why this may not be so clear in practice.)

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- Worse news: it does not only affect factoring, but also DLOG!

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We need new hardness assumptions, that can't be solved with quantum computers.
We need "post-quantum" cryptography (PQC).

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- Produce secure implementations and legal standards
- Deploy in real-world systems

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Lattice-based and isogeny-based cryptography will be explained at AS Crypto on Tuesday!

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- We can add, multiply, and divide polynomials.

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- Let S be a $k \times k$ random invertible matrix, and P a $n \times n$ permutation matrix
- Given a message $\boldsymbol{m} \in \{0,1\}^k$, encode it and perturb it on t indices, using $\boldsymbol{z} \in \{0,1\}^n$ of Hamming weight t.

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The hardness assumption [McE78]

Dados t, $G^{\text{pub}} := SGP$ y $\boldsymbol{c} := \boldsymbol{m}G^{\text{pub}} \oplus \boldsymbol{z}$, recuperar $\boldsymbol{m}$

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- $\mathcal{R}_q := \mathbb{Z}_q[x]/(\phi)$,
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NTRU [HPS98]

Given $h := g/f \mod q$, recover g or f.

More PQC using polynomial rings

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Search Ring Learning With Errors (RLWE) [Reg05, SSTX09, LPR10]

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Decision Ring Learning With Errors (RLWE) [Reg05, SSTX09, LPR10]

Given $(a, b) \in \mathcal{R}_q \times \mathcal{R}_q$, guess whether $b \sim U(\mathcal{R}_q)$ o si $b = a \cdot s + e \mod q$.

PQC from multivariate quadratic equation systems

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- $q, n, m \in \mathbb{Z}$ be integers and $\mathbb{F}_q$ be the finite field of q elements,
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Multivariate Quadratic (MQ)

Given the $p_1, \ldots, p_m$ polynomials, find a solution $\boldsymbol{y}$ to the system of equations $p_1(\boldsymbol{y}) = \cdots = p_m(\boldsymbol{y}) = \boldsymbol{0} \mod q$, if it exists.

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- Even in those cases subtle difference may be introduced.

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Let’s try using this to port a DLOG primitive to RLWE.

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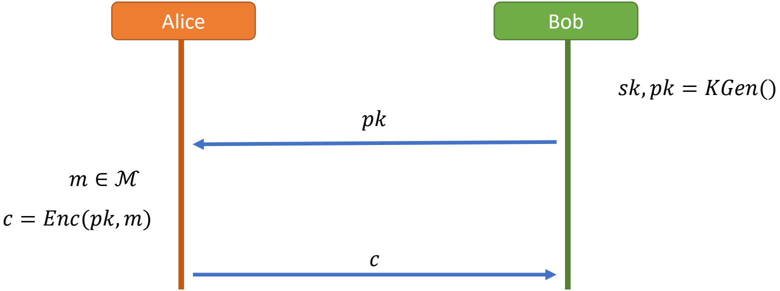

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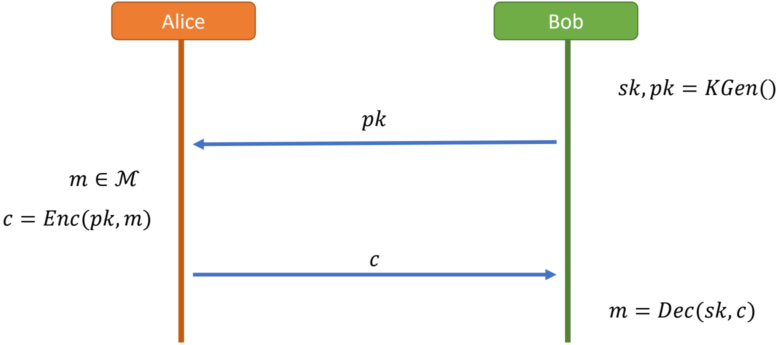

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  $= \frac{q}{2} \cdot m$ *with high probability.*

Questions so far?

Secure implementations and legal standards

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- Keep an eye on the CHES conference publications: https://tches.iacr.org

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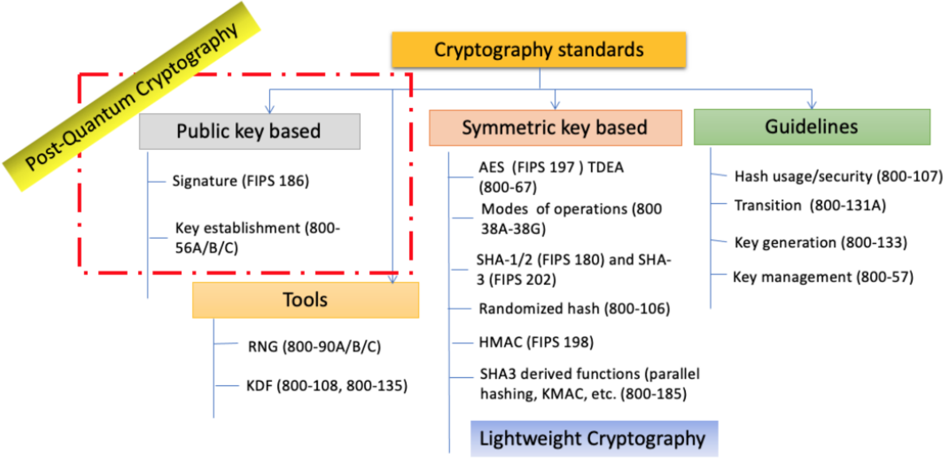

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- After multiple review rounds, in 2023 the first draft standards have been posted for comment, `https://csrc.nist.gov/projects/post-quantum-cryptography`

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- SLH-DSA: a hash-based signature scheme known as Sphincs+

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Discussions about standardisation can be followed on
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- A lot of *sensitive* code will need rewriting, with all the risks that follow! (Eg., CVE-2022-21449: Psychic Signatures in Java)

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- In the same way, schemes like Rainbow (a NIST signature scheme finalist first defined in 2005) was fully broken by Beullens in 2022 [Beu22].
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- A lot of work in cryptanalysis left to do!

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- Use hybrid schemes!
- For PKE: encrypt with EC-ElGamal, and encrypt the result with ML-KEM
- For signatures: sign with (say) EC-DSA and ML-DSA, verify *both* signatures

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Thank you

Slides @ https://fundamental.domains

Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando G. S. L. Brandao, David A. Buell, Brian Burkett, Yu Chen, Zijun Chen, Ben Chiaro, Roberto Collins, and William *et al.* Courtney.
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