Zero-knowledge Proofs

and lookup tables

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Ethereum Foundation

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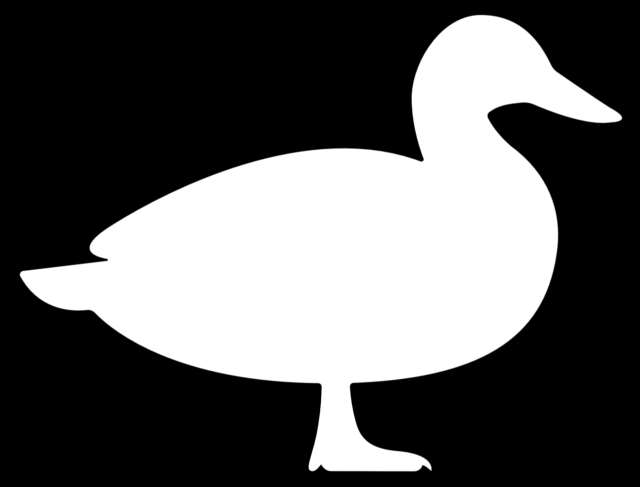
Proof Systems



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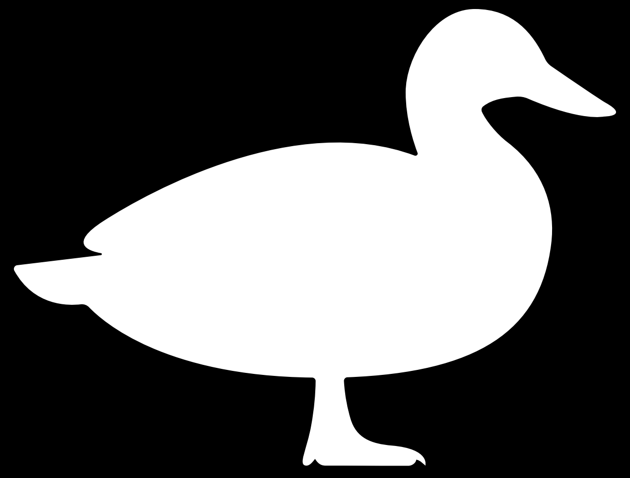

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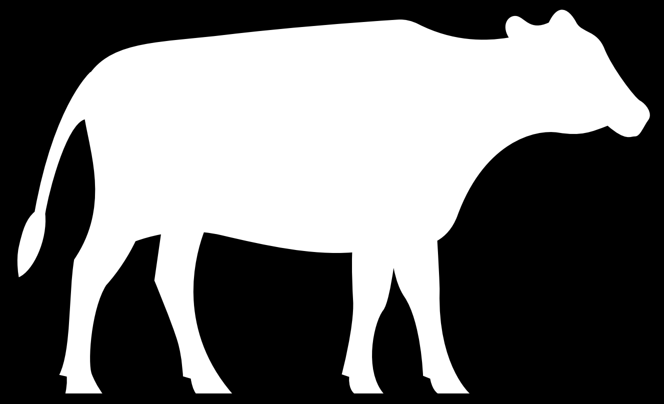

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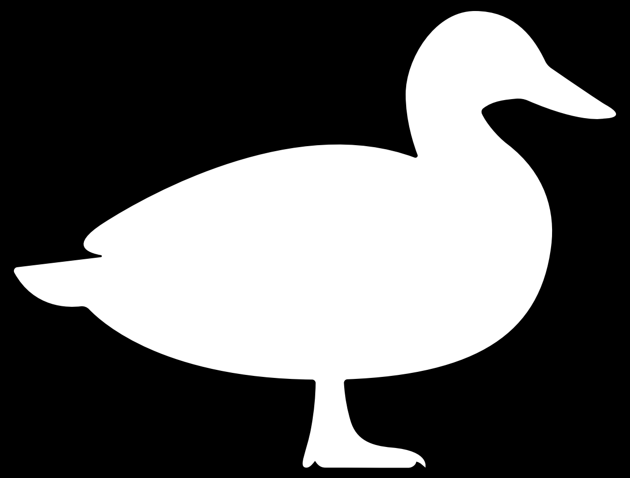

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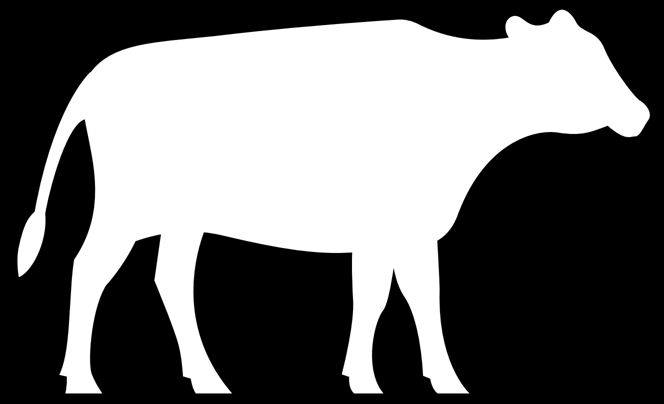

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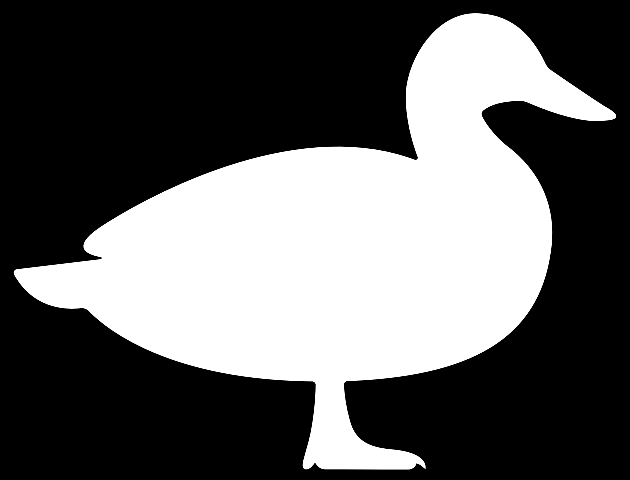

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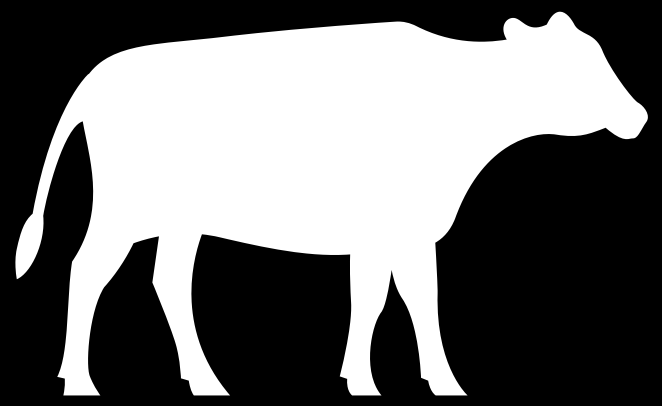
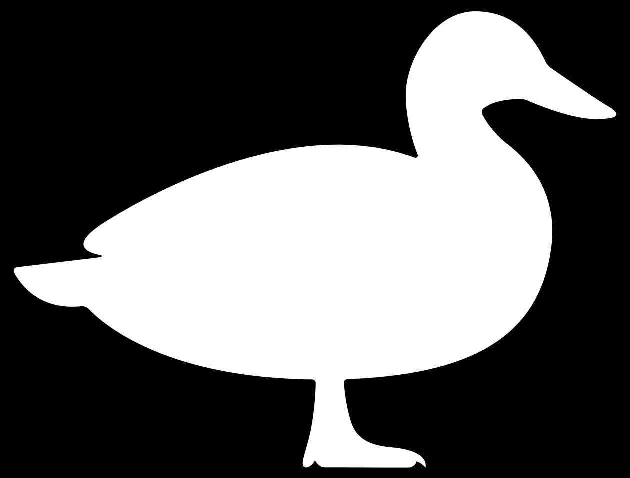


Valeria

Something is true



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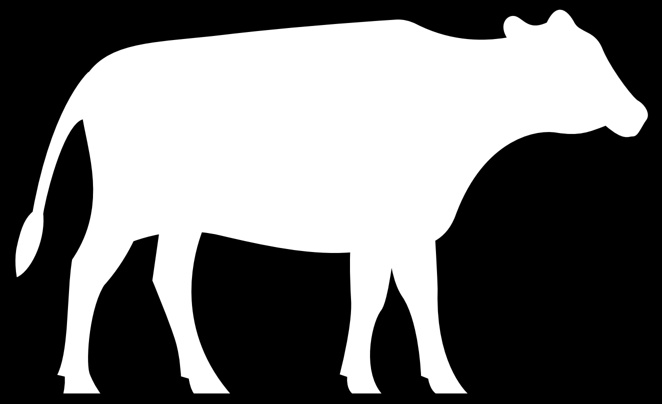
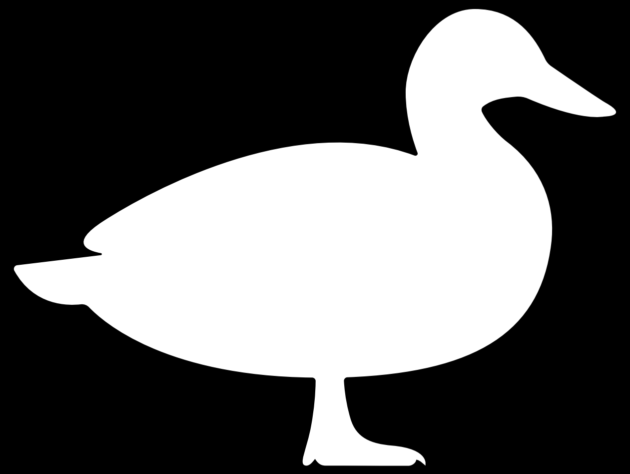


Valeria

Something is true

Something



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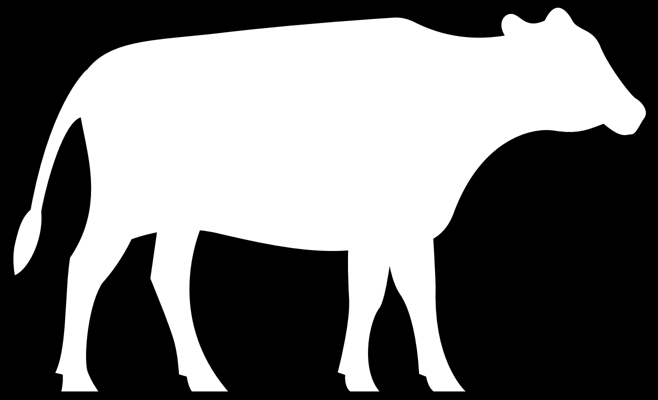
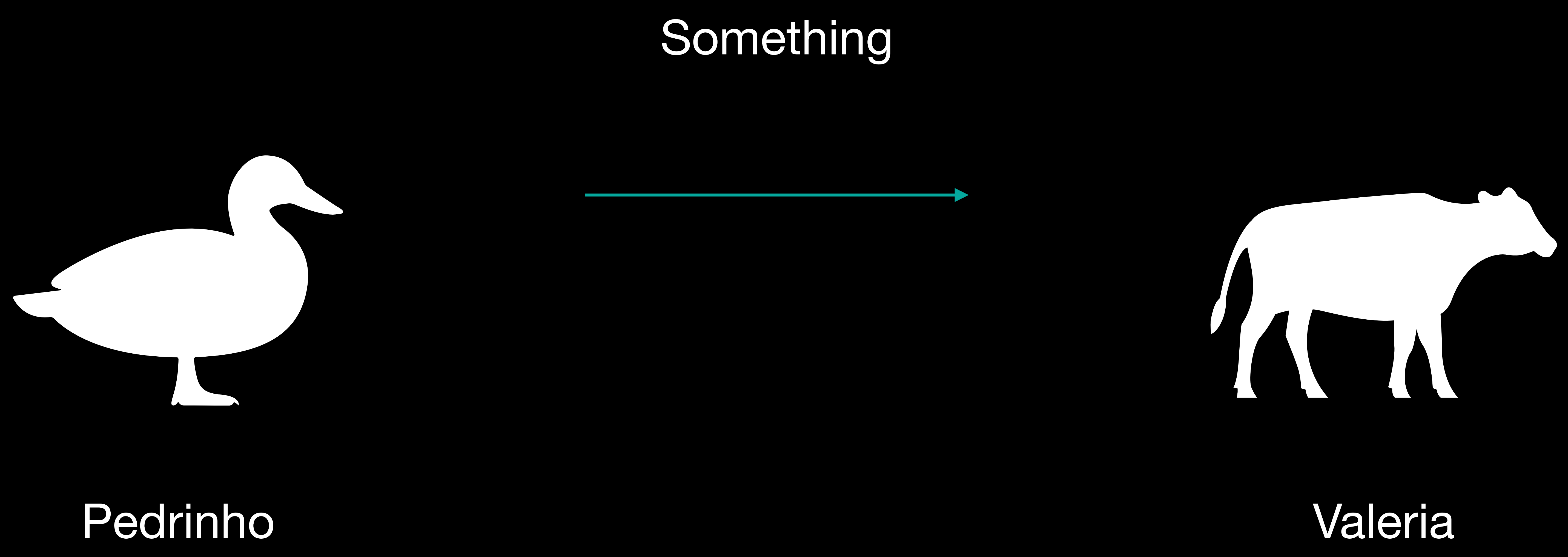


Valeria

Something is true



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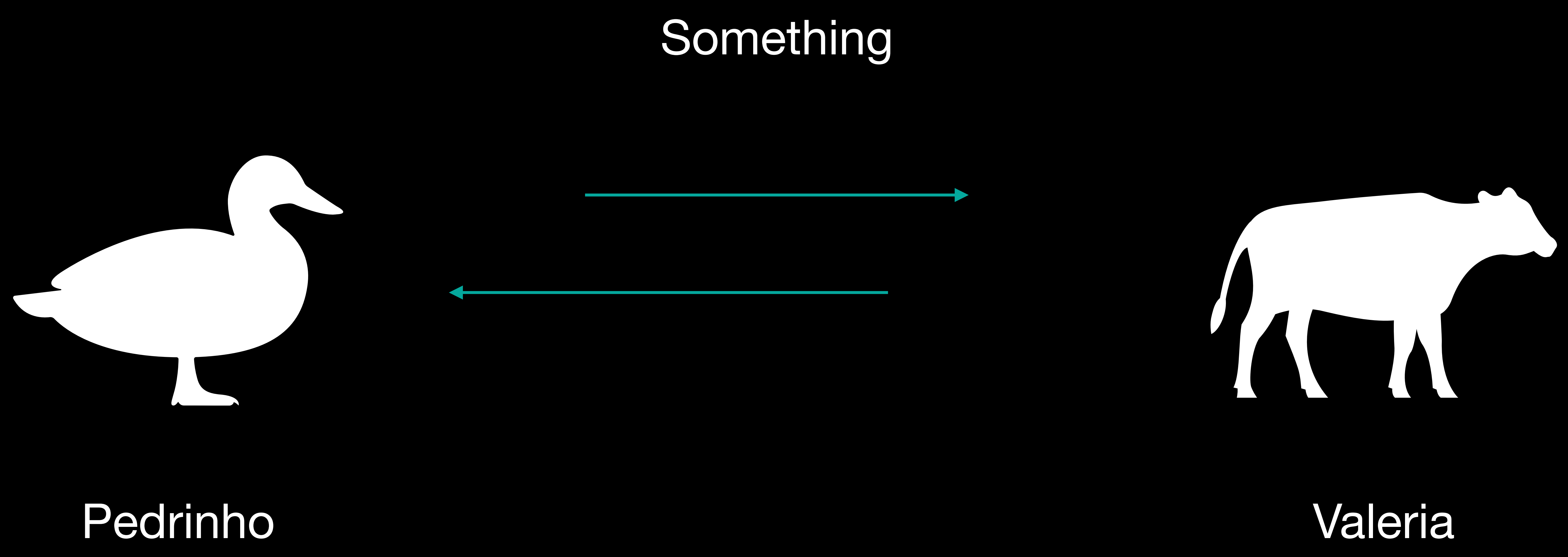
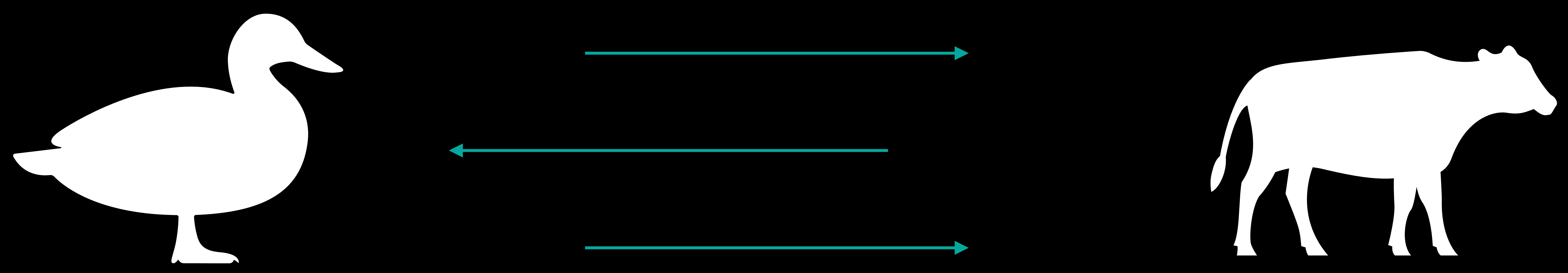


Something is true

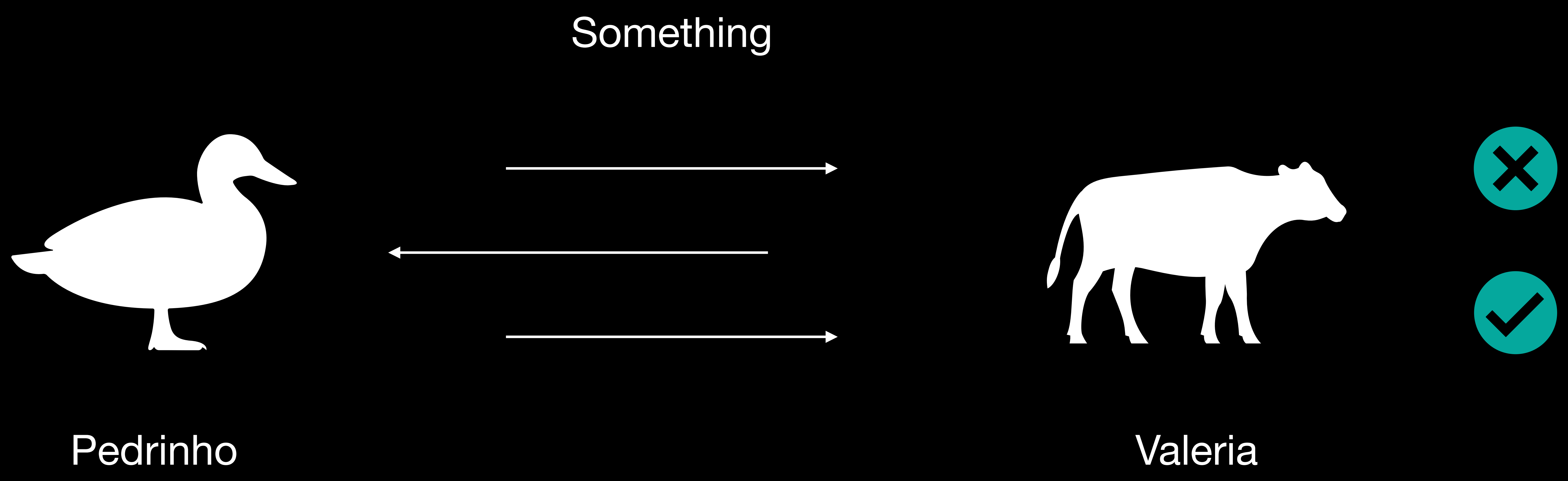
Something



Pedrinho

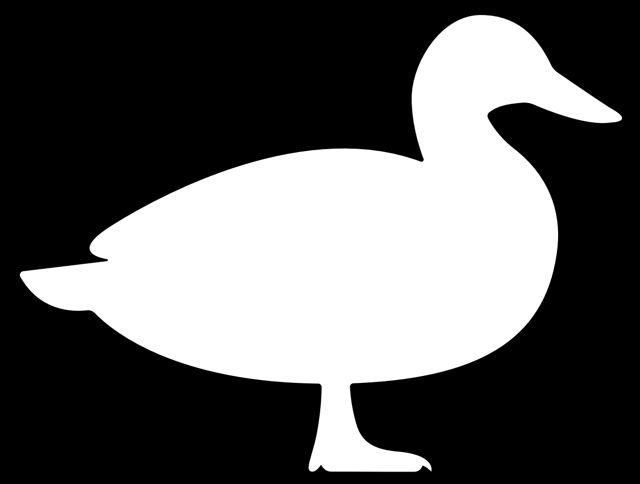
Valeria

Something is true

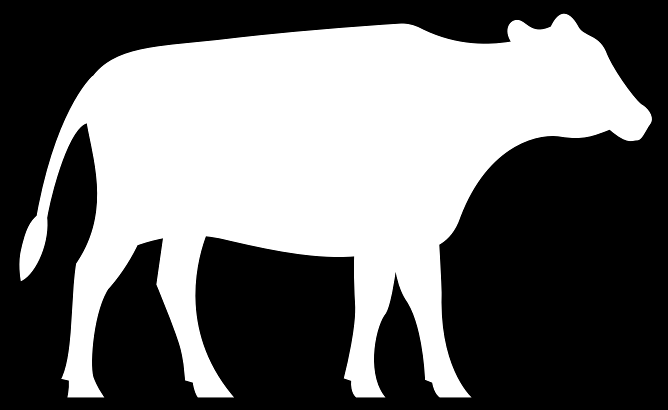


Examples of provers and verifiers

Examples of provers and verifiers

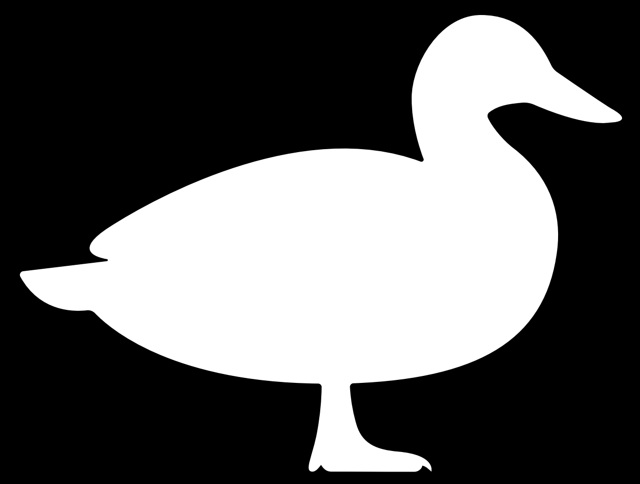


Me

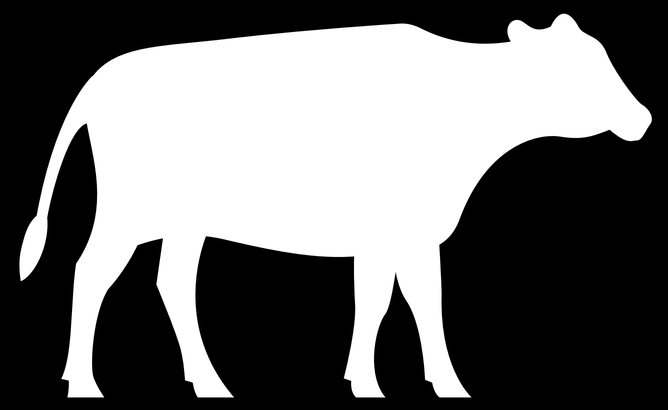


Gmail

Examples of provers and verifiers

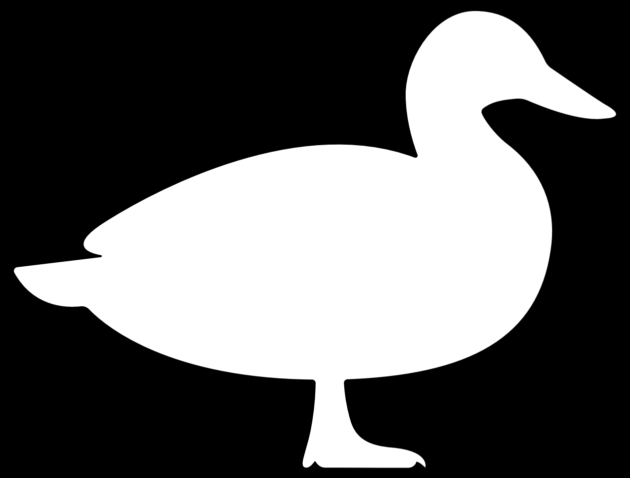


Google Cloud

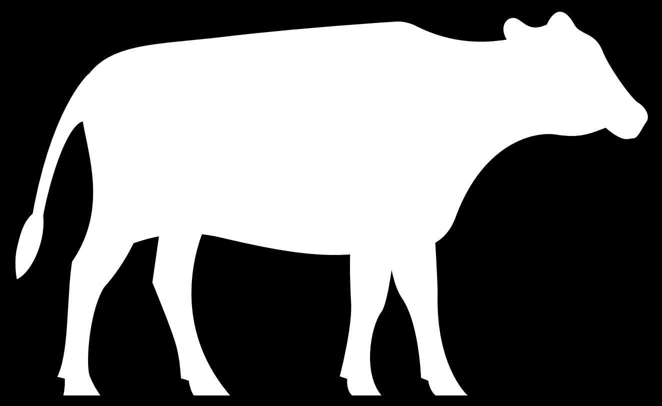


Mobil Phone

Examples of provers and verifiers

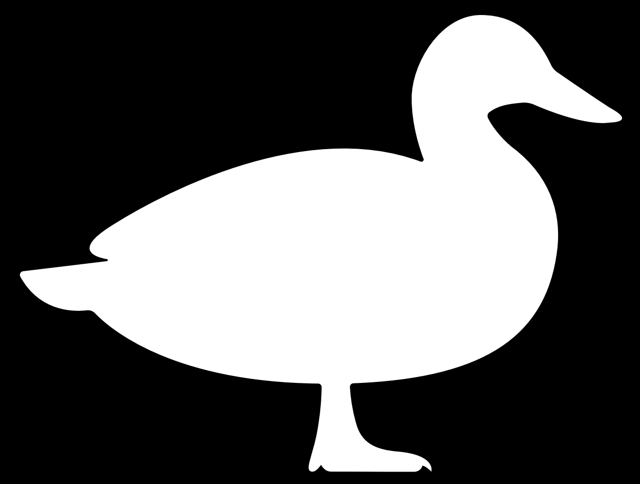


You

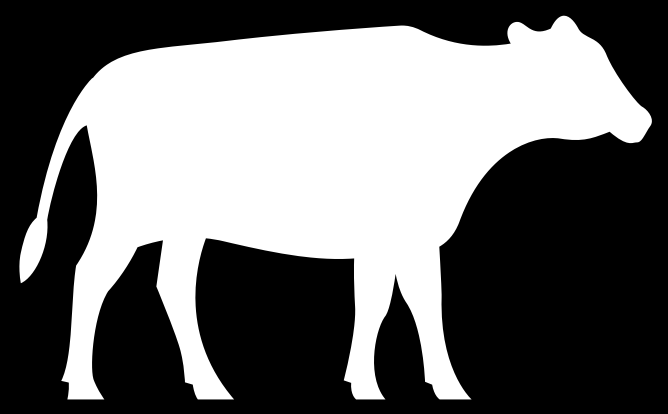


Security at Club

Examples of provers and verifiers

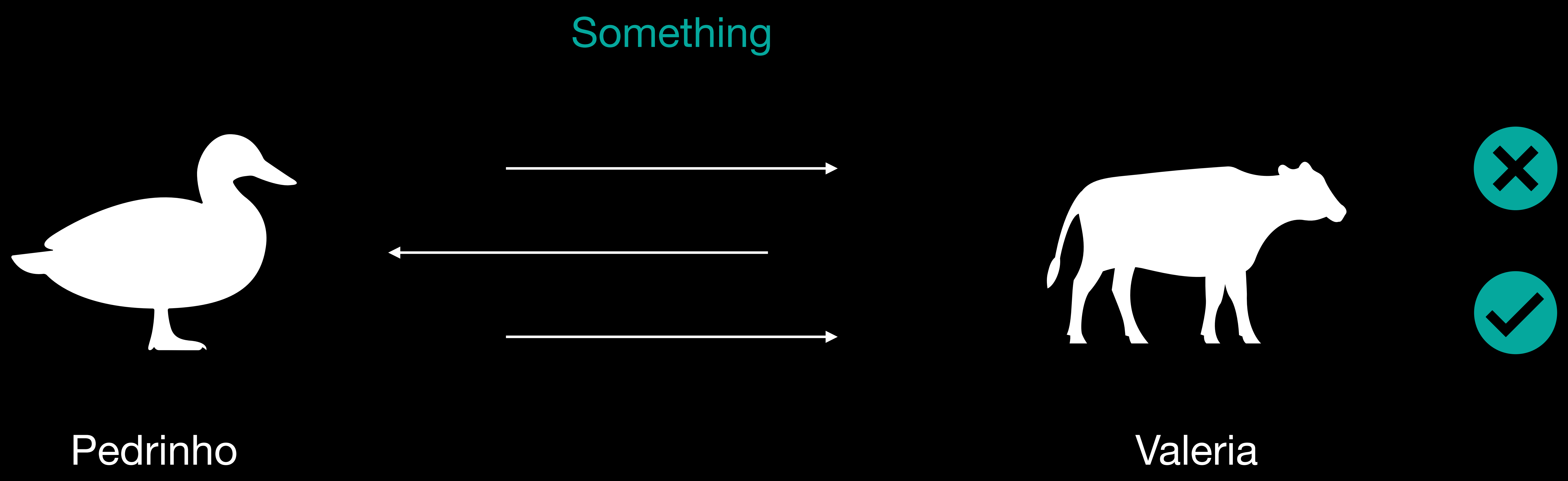


Cryptobro



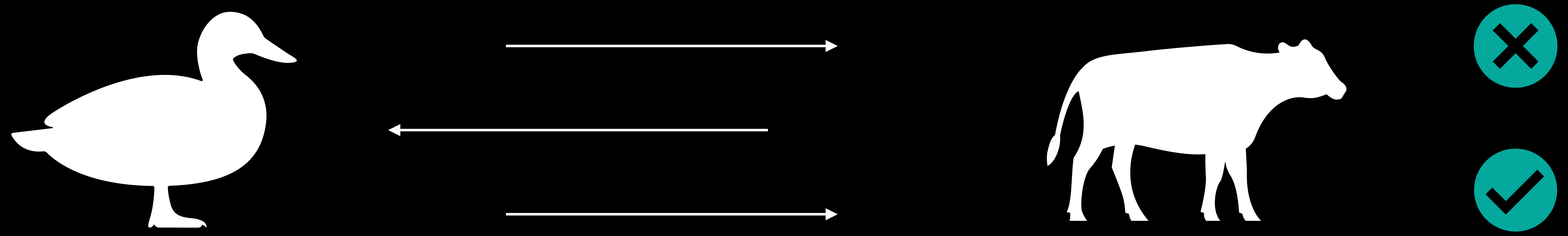
Block Builder

Something is true



Something is true

Something

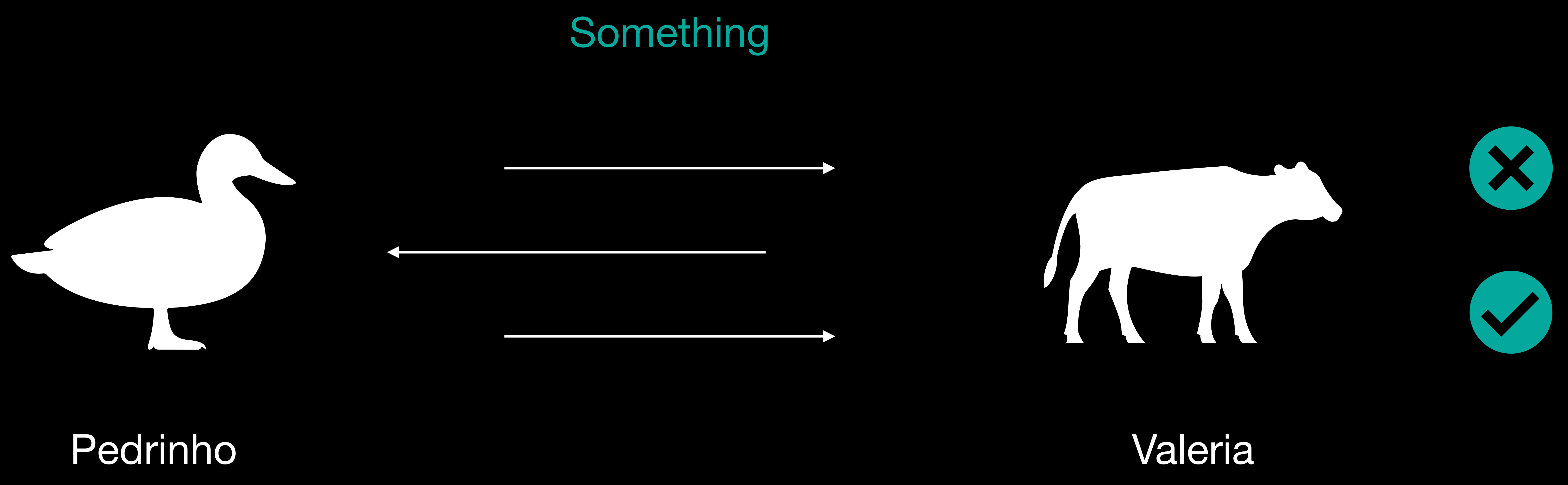


Pedrinho

Valeria

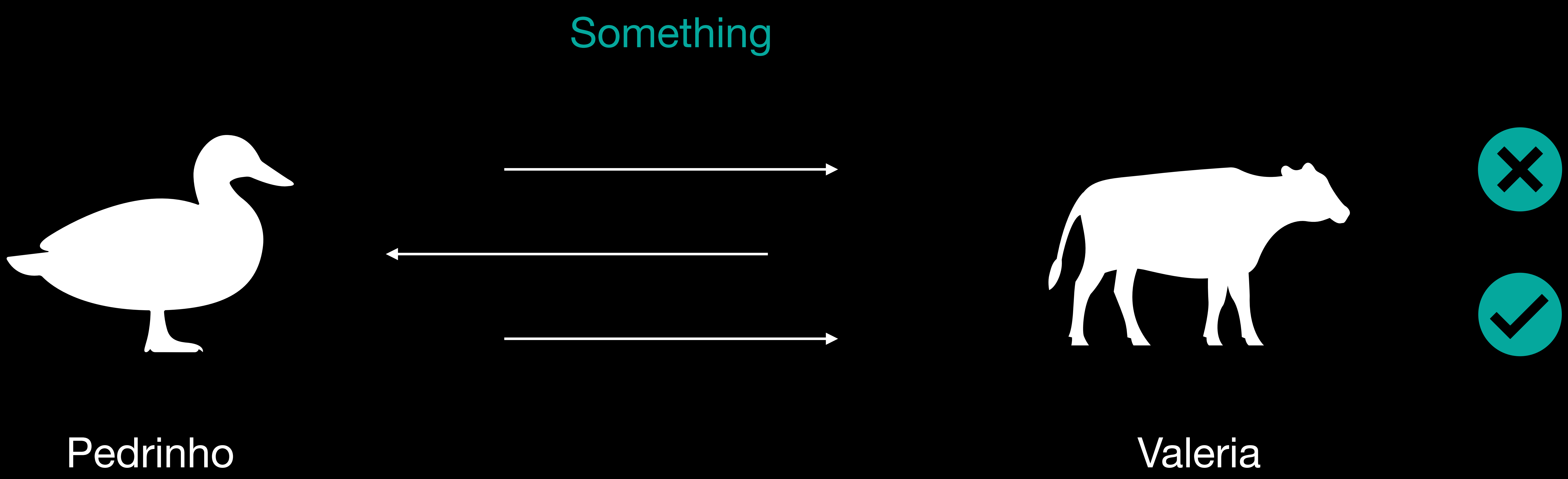
Completeness

Something is true



Completeness If Something is indeed true and both, Prover and Verifier, follow the procedure, Verifier accepts

Something is true

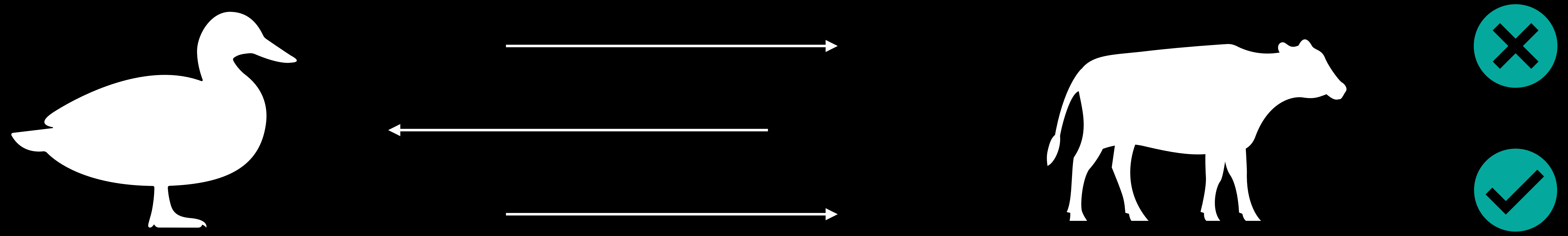


Completeness If Something is indeed true and both, Prover and Verifier, follow the procedure, Verifier accepts

Soundness

Something is true

Something



Pedrinho

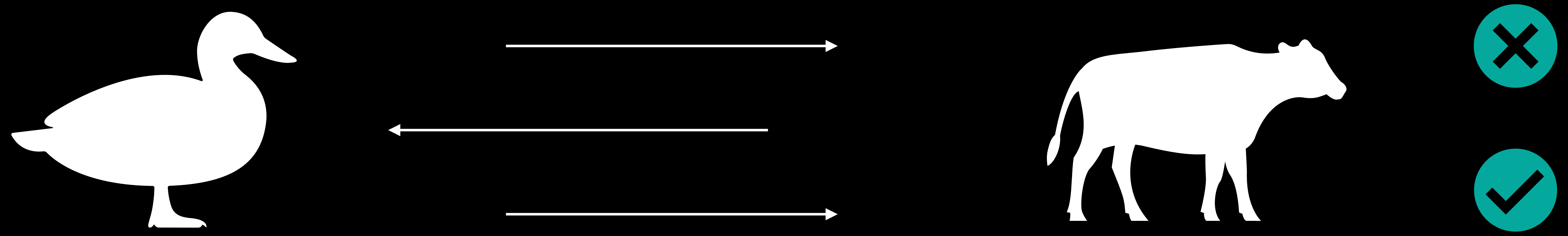
Valeria

Completeness If Something is indeed true and both, Prover and Verifier, follow the procedure, Verifier accepts

Soundness If *something* is false, then Verifier rejects with overwhelming probability

Something is true

Something



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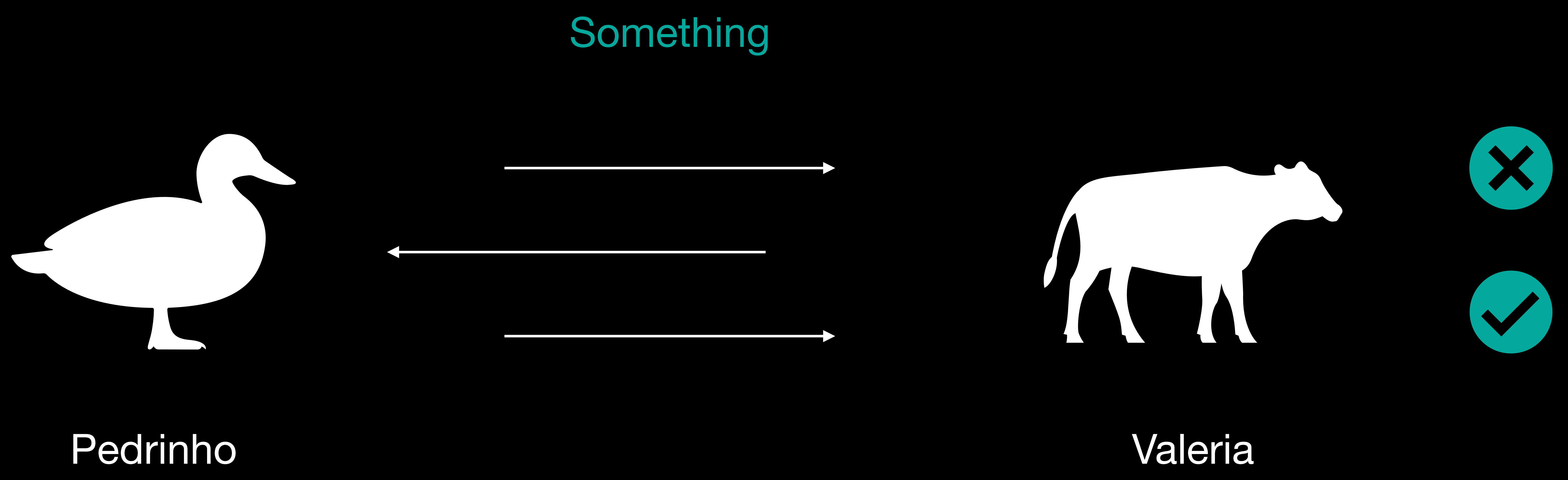
Zero-Knowledge

Something is true



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Zero-Knowledge The Verifier does not learn anything but the truth of *Something*

Something

Something

R is a PT decidable relation

$R = \{(x, w) : \ldots\}$ is a PT decidable relation

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Something is true

$R = \{(x, w) : \ldots\}$ is a PT decidable relation

$$x \in \mathscr{L}_R$$

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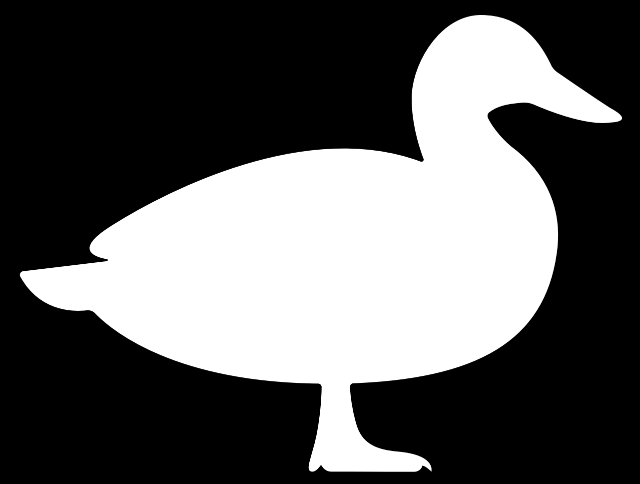
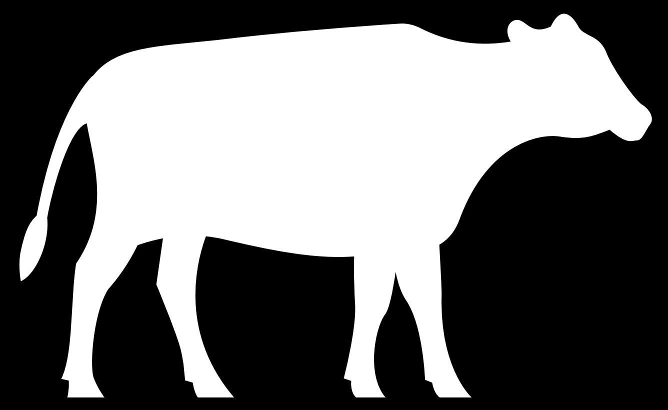
$$x \in \mathscr{L}_R$$

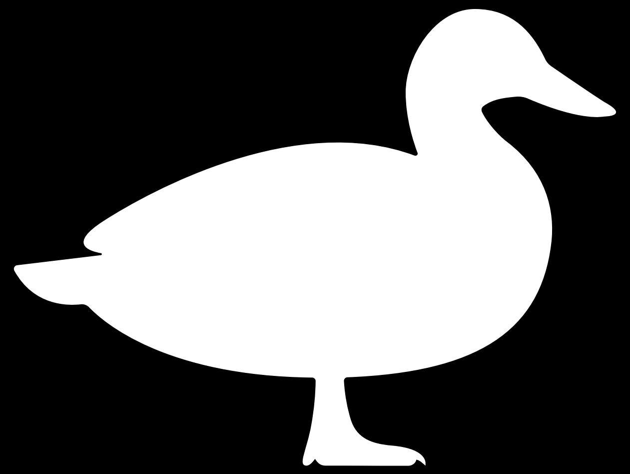
$$\mathscr{L}_R = \{x \ s.t. \ \exists w \ s.t. \ (x, w) \in R\}$$



You



Security at Club



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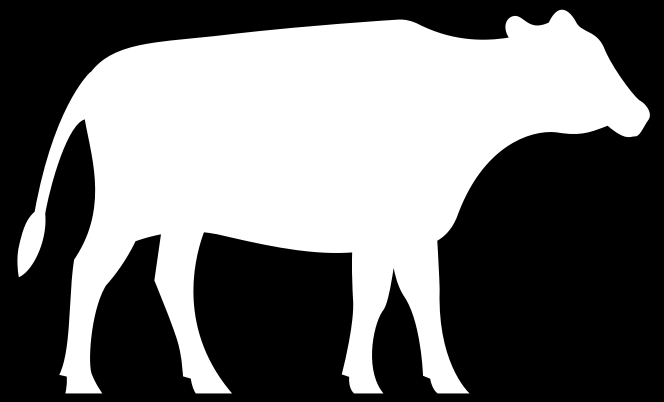
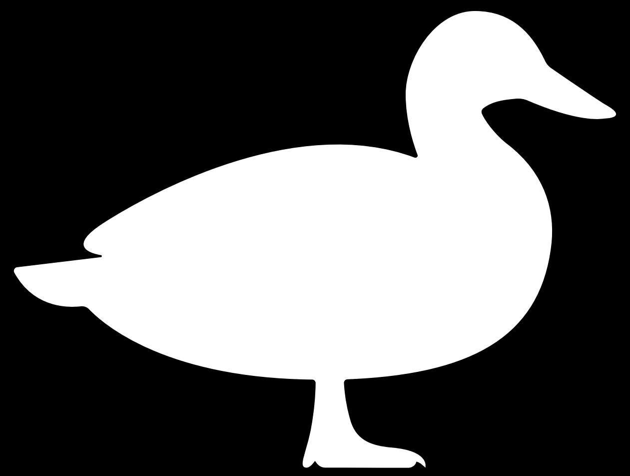


Security at Club

$$R = \{(x, w) : x \text{ is a name and } w \text{ an age above 18}$$



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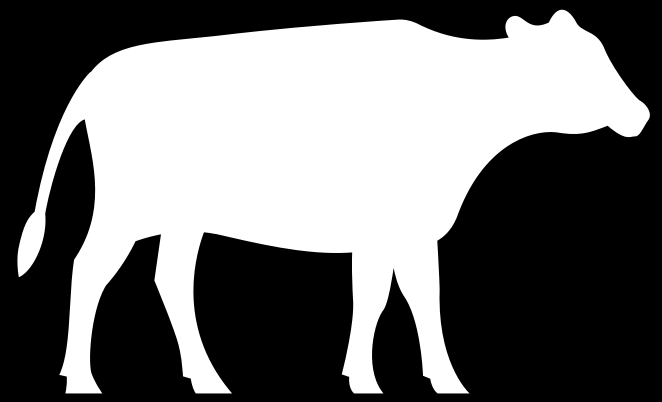
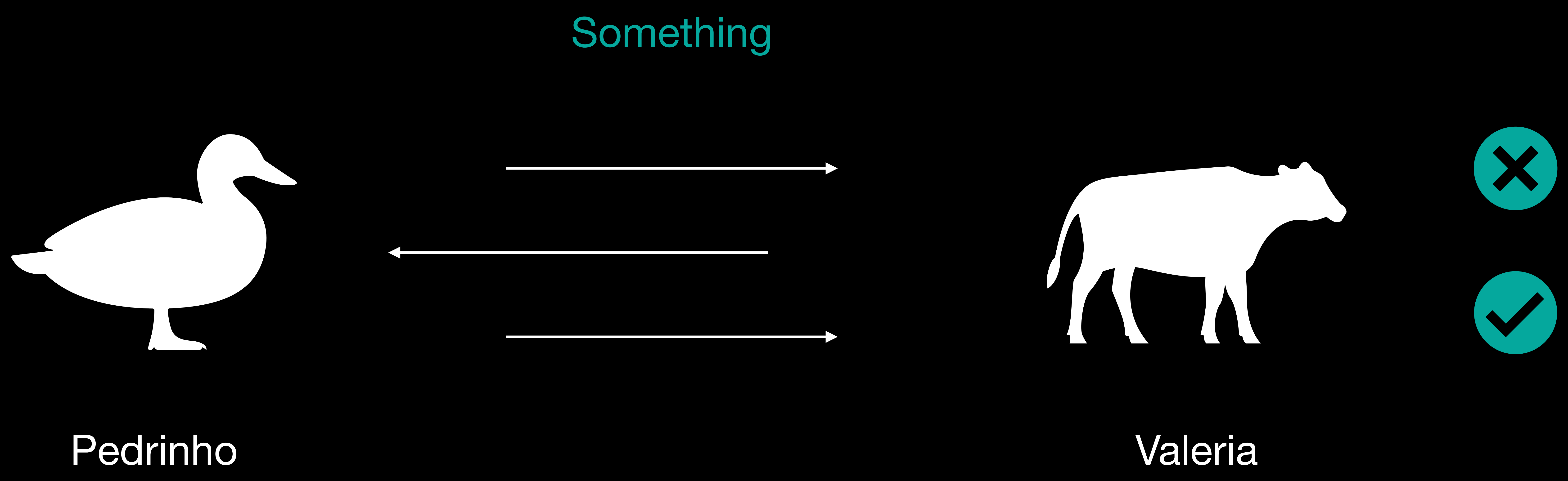


Security at Club

$$R = \{(x, w) : x \text{ is a name and } w \text{ an age above 18}$$

"I am in $\mathscr{L}_R$": there exists a w (my age) such that (me, w) $\in R$

Something is true



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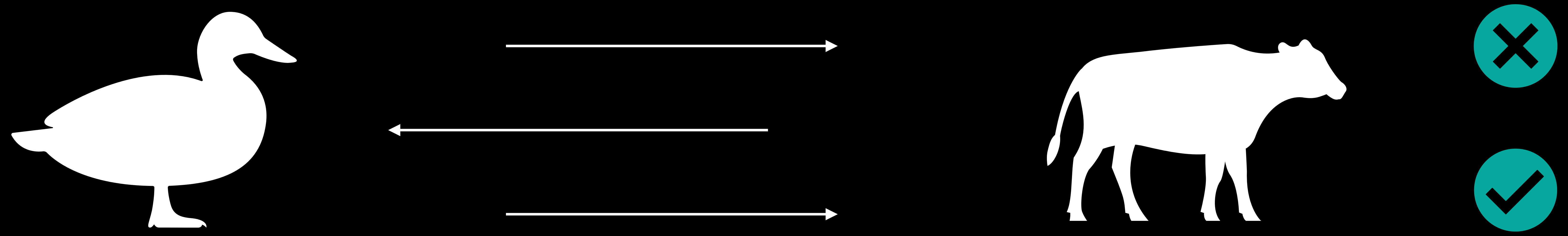
Zero-Knowledge The Verifier does not learn anything but the truth of *Something*

$$R = \{(x, w) : something\}$$



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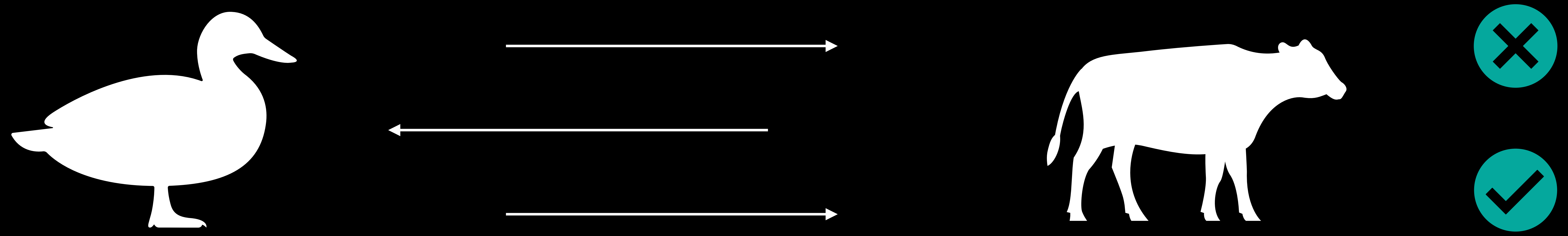
x, R



Pedrinho

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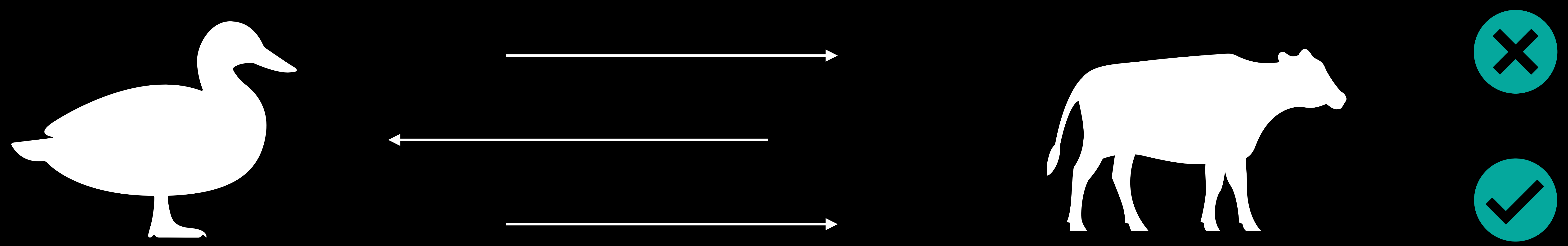

Pedrinho$((x, w), R)$ Valeria (x, R)

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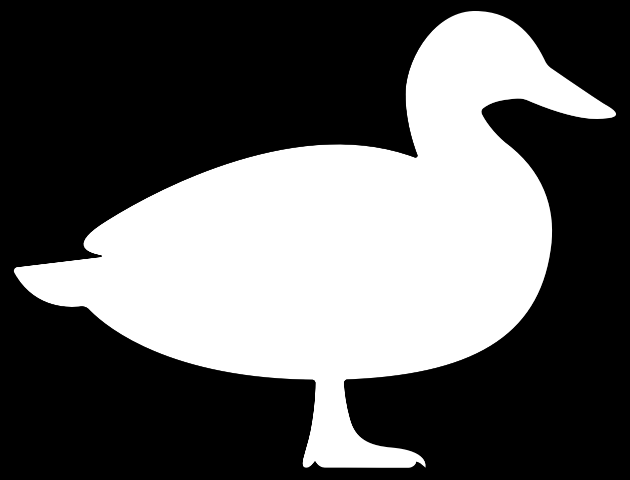

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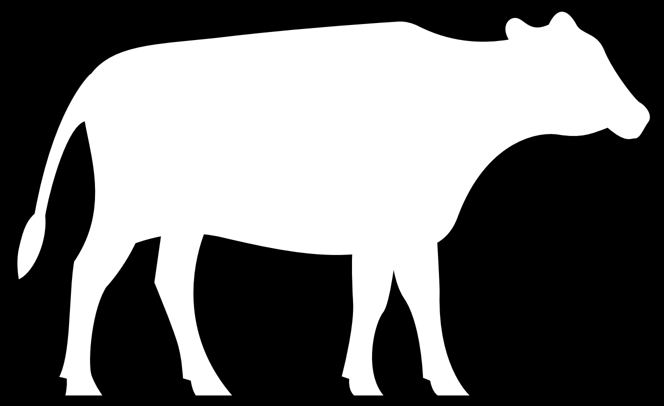
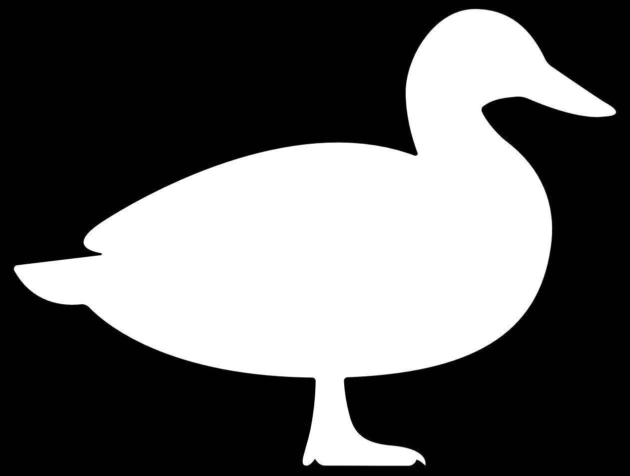


Valeria (x, R)



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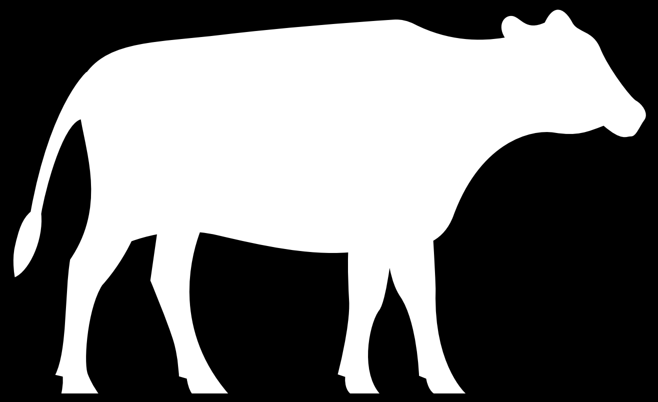

Valeria (x, R)

Probabilistic Polynomial Time Algorithms

$$\mathcal{P}$$

Prover$(srs, (x, w))$

$$\mathcal{V}$$

Verifier (srs, x)

Probabilistic Polynomial Time Algorithms

$$\mathcal{P}$$

Prover$(srs, (x, w))$

$$\mathcal{V}$$

0

1

Verifier (srs, x)

$$(srs, \tau) \leftarrow \mathcal{K}(\lambda)$$

$\mathcal{P}$

Prover$(srs, (x, w))$

$\mathcal{V}$

0

1

Verifier (srs, x)

$$R = \{(x, w) : something\}$$

srs

$\mathscr{P}$ $\longrightarrow$ $\longleftarrow$ $\xrightarrow[\pi]{}$ $\mathscr{V}$

0

1

Prover$(srs, (x, w))$

Verifier (srs, x)

$$R = \{(x, w) : something\}$$

srs

$\mathcal{P}$ $\xrightarrow{\hspace{2cm}}$ $\xleftarrow{\hspace{2cm}}$ $\xrightarrow[\pi]{\hspace{2cm}}$ $\mathcal{V}$

0

1

Prover$(srs, (x, w))$

Verifier (srs, x)

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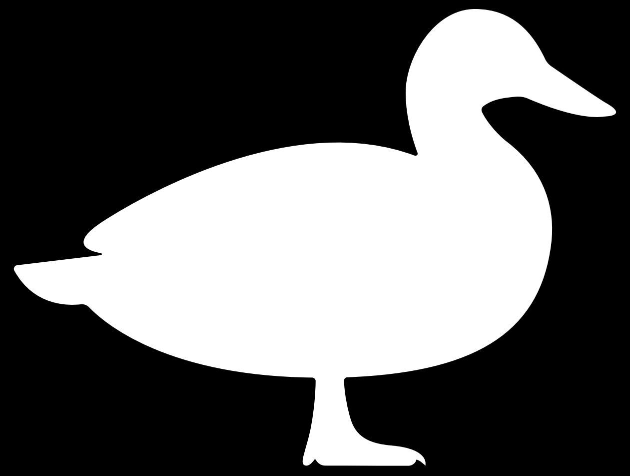
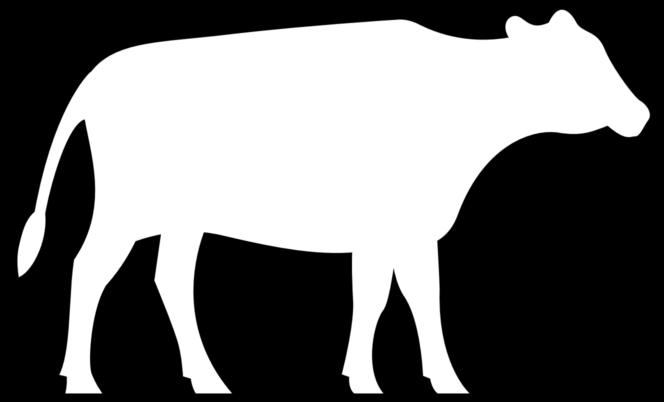
Examples of provers and verifiers

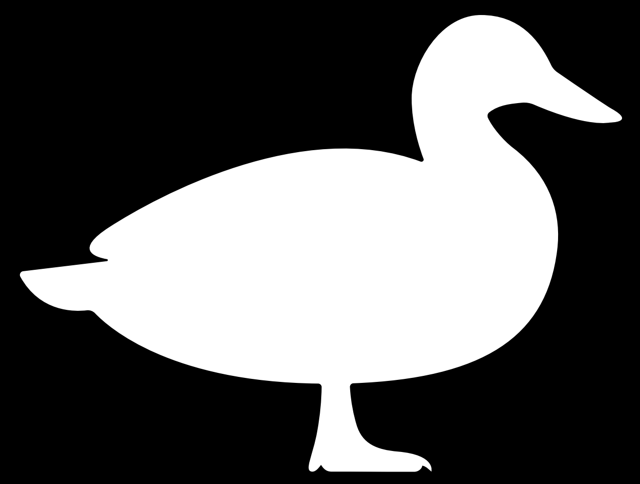


Google Cloud

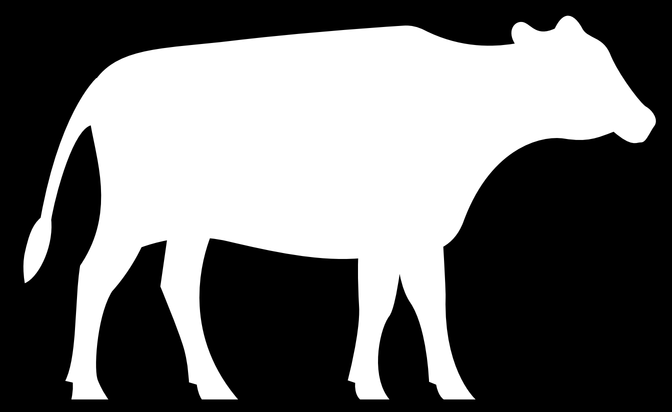


Mobil Phone

Examples of provers and verifiers



You



Security at Club

(Perfect) Completeness

(Perfect) Completeness

If Something is indeed true and both, Prover and Verifier, follow the procedure, Verifier accepts

(Perfect) Completeness

If $x \in \mathscr{L}_R$ and both, Prover and Verifier, follow the procedure, Verifier accepts

(Perfect) Completeness

If $x \in \mathscr{L}_R$ and both, Prover and Verifier, follow the procedure, Verifier accepts

$$Pr\left[\mathscr{V}(srs, x, \pi) = 1; \begin{array}{c}(srs, \tau) \leftarrow \mathscr{K}(\lambda) \\ \pi \leftarrow \mathscr{P}(srs, (x, w))\end{array}\right] = 1$$

(Perfect) Completeness

If $x \in \mathscr{L}_R$ and both, Prover and Verifier, follow the procedure, Verifier accepts

$$(srs, \tau) \leftarrow \mathscr{K}(\lambda)$$

(Perfect) Completeness

If $x \in \mathscr{L}_R$ and both, Prover and Verifier, follow the procedure, Verifier accepts

$$\pi \leftarrow \mathscr{P}(srs, (x, w))$$

(Perfect) Completeness

If $x \in \mathscr{L}_R$ and both, Prover and Verifier, follow the procedure, Verifier accepts

$$; \begin{matrix} (srs, \tau) \leftarrow \mathscr{K}(\lambda) \\ \pi \leftarrow \mathscr{P}(srs, (x, w)) \end{matrix} \Bigg]$$

(Perfect) Completeness

If $x \in \mathscr{L}_R$ and both, Prover and Verifier, follow the procedure, Verifier accepts

$$Pr \left[\mathscr{V}(srs, x, \pi) = 1; \right.$$

(Perfect) Completeness

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(Perfect) Completeness

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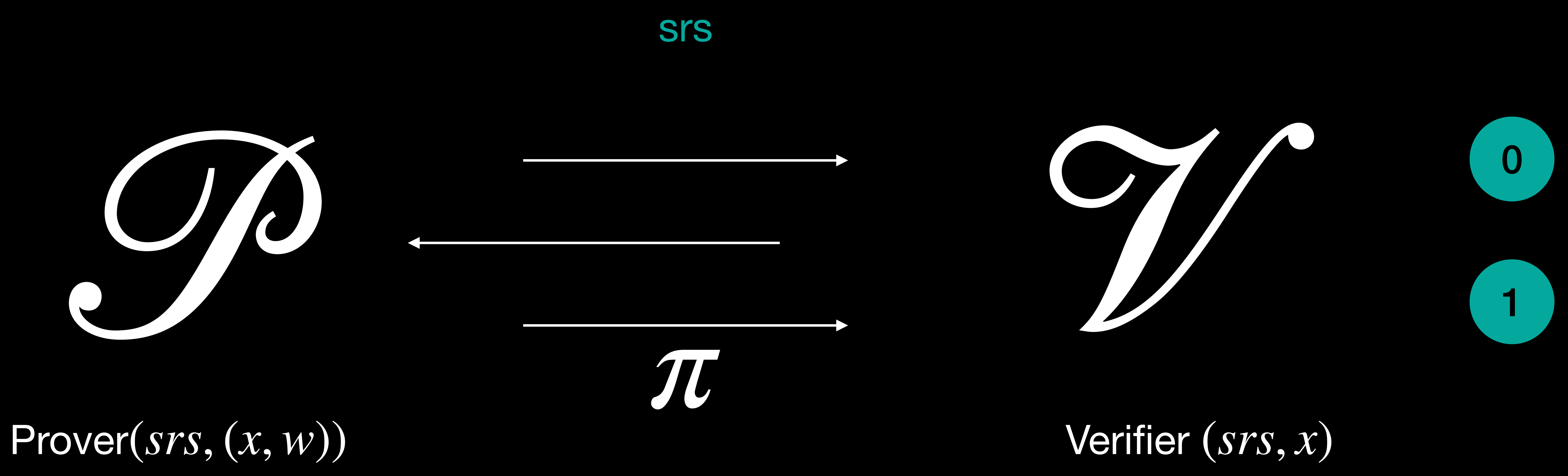
$$Pr\left[\mathscr{V}(srs, x, \pi) = 1;\ \begin{array}{r}(srs, \tau) \leftarrow \mathscr{K}(\lambda)\\ \pi \leftarrow \mathscr{P}(srs, (x, w))\end{array}\right] = 1$$

(Perfect) Completeness

If $x \in \mathscr{L}_R$ and both, Prover and Verifier, follow the procedure, Verifier accepts

$$Pr\left[\mathscr{V}(srs, x, \pi) = 1; \begin{array}{r} (srs, \tau) \leftarrow \mathscr{K}(\lambda) \\ \pi \leftarrow \mathscr{P}(srs, (x, w)) \end{array}\right] = 1$$

$$R = \{(x, w) : something\}$$



Completeness If Something is indeed true and both, Prover and Verifier, follow the procedure, Verifier accepts

Soundness If *something* is false, then Verifier rejects with overwhelming probability

Zero-Knowledge The Verifier does not learn anything but the truth of *Something*

$$R = \{(x, w) : something\}$$

srs

$$\mathscr{P} \quad \begin{array}{c} \longrightarrow \\ \longleftarrow \\ \xrightarrow[\pi]{} \end{array} \quad \mathscr{V}$$

0

1

Prover$(srs, (x, w))$

Verifier (srs, x)

Completeness

$$Pr\left[\mathscr{V}(srs, x, \pi) = 1; \begin{array}{c} (srs, \tau) \leftarrow \mathscr{K}(\lambda) \\ \pi \leftarrow \mathscr{P}(srs, (x, w)) \end{array}\right] = 1$$

Soundness

If *something* is false, then Verifier rejects with overwhelming probability

Zero-Knowledge

The Verifier does not learn anything but the truth of *Something*

(Computational) Soundness

If *something* is false, then Verifier rejects with overwhelming probability

(Computational) Soundness

If $x \notin \mathscr{L}_R$, then Verifier rejects with overwhelming probability

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If $x \notin \mathscr{L}_R$, then Verifier rejects with overwhelming probability

If $\nexists\ w\ s.t.\ (x, w) \in R$, then Verifier rejects with overwhelming probability

(Computational) Soundness

If $x \notin \mathscr{L}_R$, then Verifier rejects with overwhelming probability

If $\nexists\ w\ s.t.\ (x, w) \in R$, then Verifier rejects with overwhelming probability

$$Pr\left[\mathscr{V}(srs, x, \pi) = 1;\ \begin{matrix}(srs, \tau) \leftarrow \mathscr{K} \\ (x, \pi) \leftarrow \mathscr{A}(srs)\end{matrix}\right] \leq negl(\lambda)$$

(Computational) Soundness

If $x \notin \mathscr{L}_R$, then Verifier rejects with overwhelming probability

If $\nexists\ w\ s.t.\ (x, w) \in R$, then Verifier rejects with overwhelming probability

$$(srs, \tau) \leftarrow \mathscr{K}$$

(Computational) Soundness

If $x \notin \mathscr{L}_R$, then Verifier rejects with overwhelming probability

If $\nexists\ w\ s.t.\ (x, w) \in R$, then Verifier rejects with overwhelming probability

$$(x, \pi) \leftarrow \mathscr{A}(srs)$$

(Computational) Soundness

If $x \notin \mathscr{L}_R$, then Verifier rejects with overwhelming probability

If $\nexists\ w\ s.t.\ (x, w) \in R$, then Verifier rejects with overwhelming probability

$$Pr\Big[\mathscr{V}(srs, x, \pi) = 1;$$

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We are actually talking about arguments

$$R = \{(x, w) : something\}$$

srs

$$\mathscr{P} \quad \longrightarrow \quad \longleftarrow \quad \xrightarrow[\pi]{} \quad \mathscr{V}$$

0

1

Prover$(srs, (x, w))$

Verifier (srs, x)

Completeness

$$Pr\left[\mathscr{V}(srs, x, \pi) = 1; \begin{array}{r} (srs, \tau) \leftarrow \mathscr{K}(\lambda) \\ \pi \leftarrow \mathscr{P}(srs, (x, w)) \end{array}\right] = 1$$

Soundness

If *something* is false, then Verifier rejects with overwhelming probability

Zero-Knowledge

The Verifier does not learn anything but the truth of *Something*

$$R = \{(x, w) : something\}$$

srs

$$\mathscr{P} \quad \begin{matrix} \longrightarrow \\ \longleftarrow \\ \longrightarrow \\ \pi \end{matrix} \quad \mathscr{V}$$

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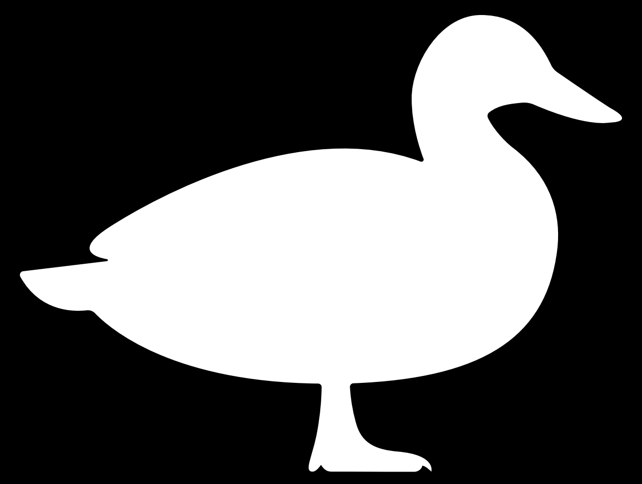
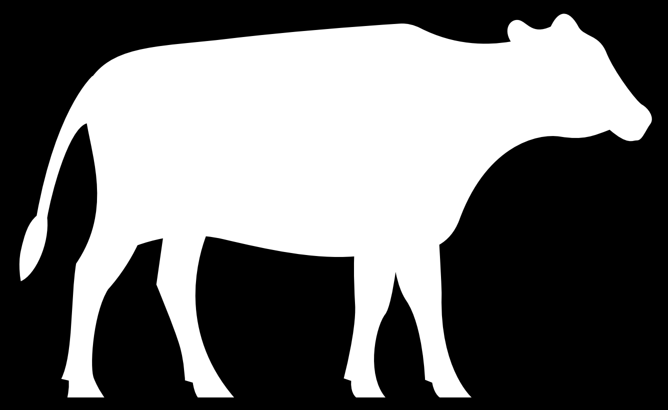
Examples of provers and verifiers



Me



Gmail

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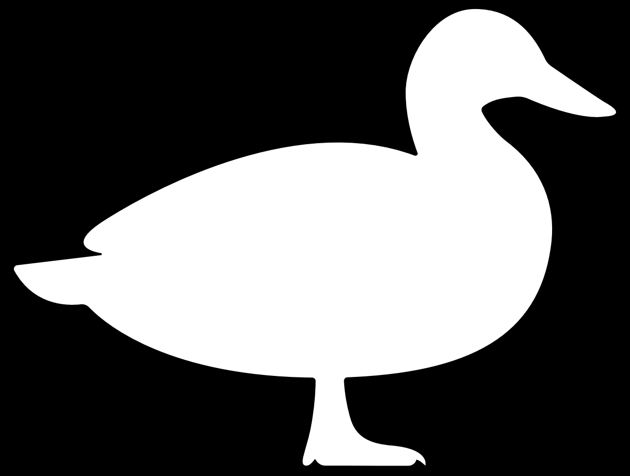

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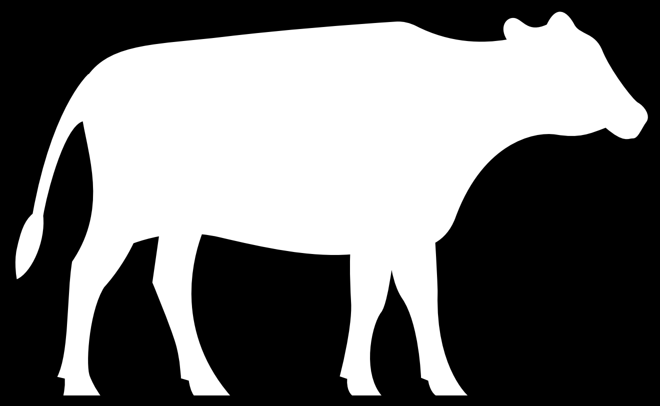

Gmail

There exists a password for this email address

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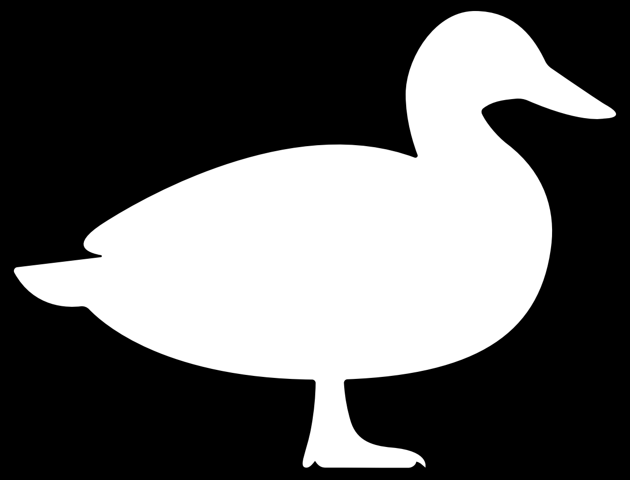
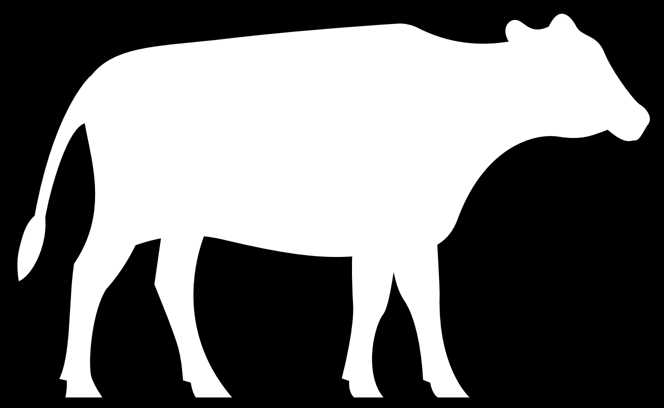


Me



Gmail

There exists a password for this email address

Not enough!!!
I should *know* it

Knowledge-soundness

There exists a PT algorithm $\mathscr{E}$, the extractor, such that for every malicious prover $\mathscr{P}^*$:

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An argument that satisfies knowledge-soundness is an **argument of knowledge**

$$R = \{(x, w) : something\}$$

srs

$$\mathscr{P} \quad \begin{array}{c} \longrightarrow \\ \longleftarrow \\ \xrightarrow[\pi]{} \end{array} \quad \mathscr{V}$$

0

1

Prover$(srs, (x, w))$

Verifier (srs, x)

Completeness

$$Pr\left[\mathscr{V}(srs, x, \pi) = 1; \begin{array}{r} (srs, \tau) \leftarrow \mathscr{K}(\lambda) \\ \pi \leftarrow \mathscr{P}(srs, (x, w)) \end{array}\right] = 1$$

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Zero-Knowledge The Verifier does not learn anything but the truth of *Something*

Zero-knowledge

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The prover output is *almost random,* therefore, could be anything

Zero-knowledge

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$$R = \{(x, w) : something\}$$

srs

$$\mathscr{P} \qquad \xrightarrow{\hspace{2em}} \qquad \mathscr{V}$$

$$\xleftarrow{\hspace{2em}}$$

$$\xrightarrow[\pi]{\hspace{2em}}$$

0

1

Prover$(srs, (x, w))$ — Verifier (srs, x)

Completeness $$Pr\left[\mathscr{V}(srs, x, \pi) = 1; \begin{array}{c} (srs, \tau) \leftarrow \mathscr{K}(\lambda) \\ \pi \leftarrow \mathscr{P}(srs, (x, w)) \end{array}\right] = 1$$

Knowledge-Soundness $$Pr\left[\begin{array}{c} (x, w) \notin R \wedge \\ \mathscr{V}(srs, x, \pi) = 1 \end{array}; \begin{array}{c} (srs, \tau) \leftarrow \mathscr{K} \\ (x, \pi) \leftarrow \mathscr{A}(srs) \\ w \leftarrow \mathscr{E}(srs, x, \pi) \end{array}\right] \leq negl(\lambda)$$

Zero-Knowledge The Verifier does not learn anything but the truth of *Something*

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srs

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Everything that can be proven (NP) can be proven in Zero-Knowledge

Knowledge of discrete log

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Discrete logs are hard to compute
(In some groups)

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Let $\mathbb{G}$ be a cyclic group of order q (prime) and g be a generator.

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Famous secret-key, public-key couple:

$$sk \leftarrow \mathbb{Z}_q, \quad pk = g^{sk}$$

Knowledge of discrete log - Schnorr

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$\mathcal{P}$

$\mathcal{V}$

Knowledge of discrete log - Schnorr

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$((h, \mathbb{G}), (x, h))$

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$((g, \mathbb{G}), h)$

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$$((h, \mathbb{G}), (x, h)) \qquad\qquad ((g, \mathbb{G}), h)$$

$$\begin{array}{ccc} r \leftarrow \mathbb{Z}_q & & \\ u = g^r & \xrightarrow{\quad u \quad} & \\ & & c \leftarrow \mathbb{Z}_q \\ & \xleftarrow{\quad c \quad} & \\ z = r + cx & & \end{array}$$

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$g^z = uh^c$

Completeness

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Theorem: The scheme satisfies completeness

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With probability ϵ^2, $g^{z_1} = uh^{c_1} \;\wedge\; g^{z_2} = uh^{c_2}$. Then,

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$$\boxed{\frac{g^{z_1}}{h^{c_1}} = \frac{g^{z_2}}{h^{c_2}} \rightarrow \frac{g^{z_1}}{g^{z_2}} = \frac{h^{c_1}}{h^{c_2}} \rightarrow g^{z_1 - z_2} = h^{c_1 - c_2} \rightarrow g^{z_1 - z_2} = (g^x)^{(c_1 - c_2)} \rightarrow g^{\frac{z_1 - z_2}{c_1 - c_2}} = (g^x)}$$

Honest-Verifier Zero-knowledge

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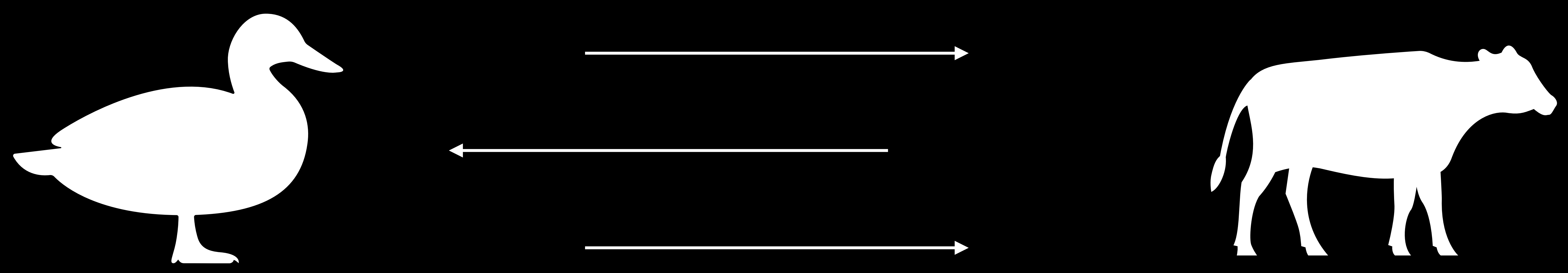
- $c \leftarrow \mathbb{Z}_q$
- $u = \dfrac{g^z}{h^c}$
- Output (u, c, z)

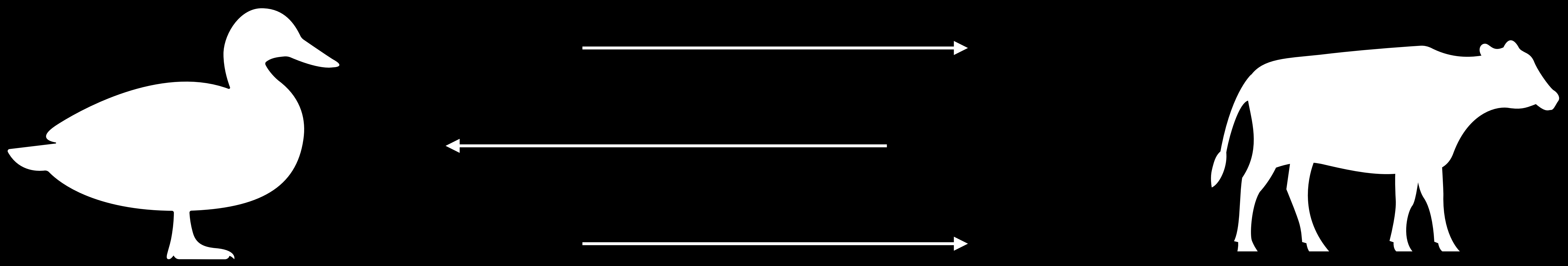
$$g^z = uh^c$$

Lookup Tables



Pedrinho Valeria

$$\overrightarrow{T} = (v_1, v_2, v_3, \ldots, v_m)$$



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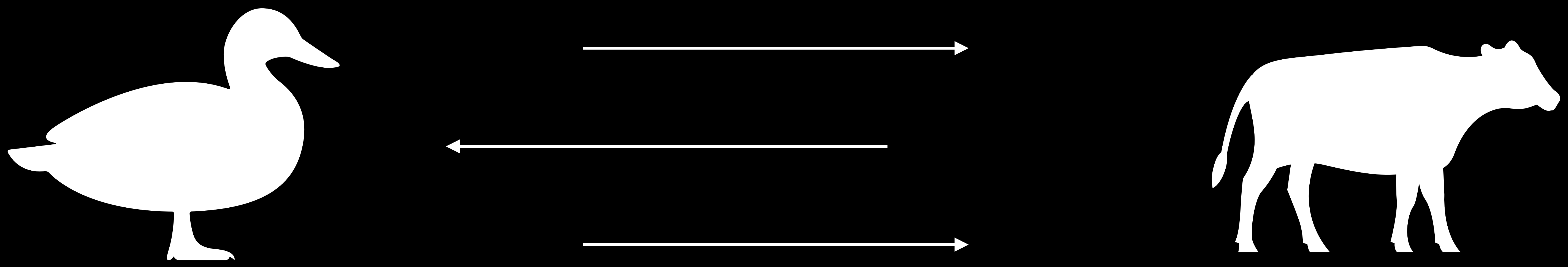
C *is a commitment to elements* $s_i \in \overrightarrow{T}$



Pedrinho Valeria

Importance

- Building blocks to many systems
- Efficiency: mostly do not depend of the size of the table
- Flexibility: zero-knowledge/succinctness/pre-computable

Some examples

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Membership proofs from Lookup tables

C *is my user name*

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Membership proofs from Lookup tables

I am an authorized member/
my name is on the list

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3. *It is me!*

¡¡¡Gracias!!!

Obrigado!!

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