Testudo

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What is a SNARK?

Prover

Let F be a 2-input function, and x a public input

∃ w such F(x,w)=y

Verifier

Π

Properties of SNARKs

- If computing **F** takes **T** prover should be **~O(T)**
- Π should be short
- Verifier should run in **o(T)**

Requirements for practical SNARKs

- linear or quasilinear prover
- logarithmic proof size (or smaller)
- logarithmic verifier (or smaller)

In many cases:

- verification happens on-chain so every operation matters 💰

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In many cases:

- verification happens on-chain so every operation matters 💰
    - **Groth16** (an improved version of **G**GPR13) still remains
        - the cheapest proof system to verify with constant number of operations
        - smallest proof size: constant
            - < 200 bytes
            - exact size depends on the curve

But we’re not fully happy with Groth16… 😕

- *function-specific* trusted setup
  - any updates to the circuit require a new ceremony *which we want to avoid at all costs*
- trusted setup is linear in the size of the circuit
  - for large circuits ⇒ requires **GBs** of storage
  - **limits** the *maximum* size of a circuit
- Groth16 uses FFTs on the prover’s side
  - hard to parallelise
  - put strain on the prover for large circuits due to memory requirements

On the other hand

- *Transparent SNARKs*
  - Have no trusted setup (Spartan, Starks)
- *Universal setup SNARKs*
  - Have a trusted setup that depends only on the size of the circuit
  - Plonk, Hyperplonk, etc.
- However those have longer proofs and verification time than Groth16
  - But usually a faster prover

Let's build a SNARK with the same proof size and verification cost as Groth16, but also a *universal* and *small* trusted setup!

General idea of **Testudo**

- Prover wants to prove that ***F(x,w)=y***
  - Run a fast transparent SNARK **P** that the prover knows ***w***
    - to get a long proof ***Π***
  - Then run a Groth16 proof that the prover knows a correct ***Π***
    - That satisfies the verification algorithm ***V(Π)=1***
    - This yields a shorter (constant) proof *Π'*
- Trusted setup is short and universal
  - The computation over which we run Groth16 is the verification algorithm of the transparent SNARK
    - Which is logarithmic in the size of ***F***
    - And works for any ***F***
- Ideas inspired by other 2-level recursion works [Belling et al. '22]
  - Independently ZKBridge developed a similar approach
  - Specific computations, we are the first to use it for generic computations

Technical contributions

- We started with Spartan as the underlying transparent SNARK
  - But that has $O(\sqrt{N})$ proofs and we wanted shorter proofs to feed into Groth16
- Changed Spartan to use a new polynomial commitment for the witness
  - Started with the PST'13 polynomial commitment for multivariate polynomials
    - Log-size proofs
    - But that requires a $O(N)$ trusted setup (where N is the size of the R1CS)
      - Too long for us
- Modified PST'13 to achieve $O(\sqrt{N})$ trusted setup
  - Using ideas from the MIPP protocol [B+21]
- Implemented this over a cycle of curve to achieve greater efficiency

Refresher: rank-1 constraint system (R1CS)

- R1CS generalises the circuit satisfiability problem
- Consider finite field $\mathbb{F}$, matrices A, B, C and vector z
  - A, B, C model the actual circuit and are public
  - z is the private witness
- An R1CS instance is satisfiable, iff the following relation holds

$$(A \cdot z) \circ (B \cdot z) = C \cdot z$$

Why R1CS

- Introduced as *Quadratic Span Programs* in **G**GPR13
  - A form of arithmetization of generic computations
  - Groth16 and Spartan uses R1CS to generate proofs
- There are alternative ways to encode a computation
  - E.g. higher order constraints a la Plonk
- Motivation came from improving the SNARK used by the Filecoin protocol
  - Filecoin relies on storage providers proving that they are committing a certain amount of memory to the network
  - Uses a large *Proof of Space* too long to post on chain
  - The Groth16 SNARK is used to compress it to constant size
    - Run over an R1CS of size about 2^30
- Building a new R1CS based SNARK would give us a “drop-in” replacement for Groth16 in Filecoin

Spartan

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- Recall $(A \cdot z) \circ (B \cdot z) = C \cdot z$ tells whether R1CS instance is satisfiable

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- Represent ***matrices as functions***

  $A, B, C : \{0,1\}^{logn} \times \{0,1\}^{logn} \rightarrow \mathbb{F}$ and $z : \{0,1\}^{logn} \rightarrow \mathbb{F}$

  $\downarrow$

$A(i,j) = A_{i,j}$

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- Using MLE extensions over the boolean hypercube

$$F(x) = \sum_{y \in \{0,1\}^n} \tilde{A}(x,y) \cdot \tilde{z}(y) \times \sum_{y \in \{0,1\}^n} \tilde{B}(x,y) \cdot \tilde{z}(y) - \sum_{y \in \{0,1\}^n} \tilde{C}(x,y) \cdot \tilde{z}(y)$$

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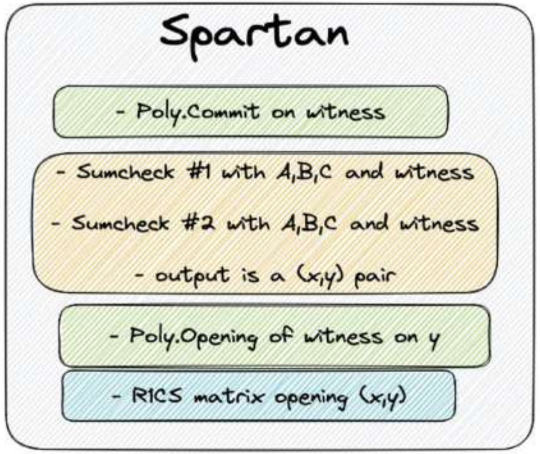
- Then, we can define the following sumcheck instance

$$0 = \sum_{x \in \{0,1\}^n} \tilde{eq}(x,t) \cdot F(x)$$

↓

1 if $x = t$ otherwise 0

Spartan



- **V** ask for evaluation at (r_y, r_x)
- P
  - commits to witness $z(y)$ with polynomial commitment scheme
    - proof size and verification between and $O(\sqrt{n})$ and $O(\log(n))$
      - depends on whether we allow a trusted setup
  - uses *computation commitments* for $\tilde{A}(x,y), \tilde{B}(x,y), \tilde{C}(x,y)$
    - generating this is part of the public setup
    - commitment exploits the sparsity of the R1CS matrices
      - ensures prover time remains quasilinear (at most $O(n \log n)$)
  - responds with evaluations and proof of opening for the 4 polynomials

PST13

$p(x_1, \ldots, x_n)$ - multilinear polynomial with n where $n = \log N = \log |C|$

Trusted Setup:

$t_1, ..., t_n$ and CRS is N elements $g^{\chi_i(t)}$ where $N = 2^n$

Commitment

$$Com(p) = g^{p(\vec{t})} = C$$

Opening

for $\vec{a} = [a_1, ...a_n]$ and $y = p(\vec{a})$

$p(\vec{x}) - y = \sum_i (x_i - a_i) q_i(\vec{x})$

proof is $\vec{w} = [w_1 \ldots w_n]$ where $w_i = g^{q_i(\vec{t})}$

Verification

$$e(Cg^{-y}, g) = \Pi_i e(g^{t_i - a_i}, w_i)$$

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logarithmic proof size

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Verification

$e(Cg^{-y}, g) = \Pi_i e(g^{t_i - a_i}, w_i)$ logarithmic verification cost

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Commitment

back to a O(N) circuit-size dependent trusted setup

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$$Com(p) = g^{p(\vec{t})} = C$$

Opening

for $\vec{a} = [a_1, \ldots a_n]$ and $y = p(\vec{a})$

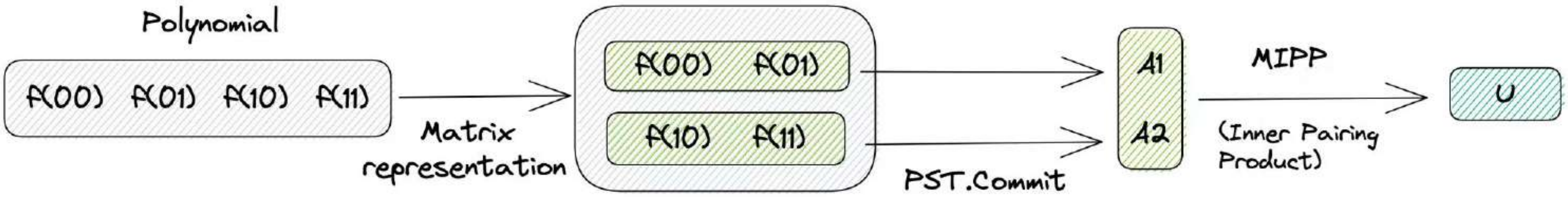
$p(\vec{x}) - y = \sum_i (x_i - a_i) q_i(\vec{x})$ *log|C|* polynomial divisions, the largest one dominates the cost 

proof is $\vec{w} = [w_1 \ldots w_n]$ where $w_i = g^{q_i(\vec{t})}$

Verification

$$e(Cg^{-y}, g) = \Pi_i e(g^{t_i - a_i}, w_i)$$

sqrt-PST

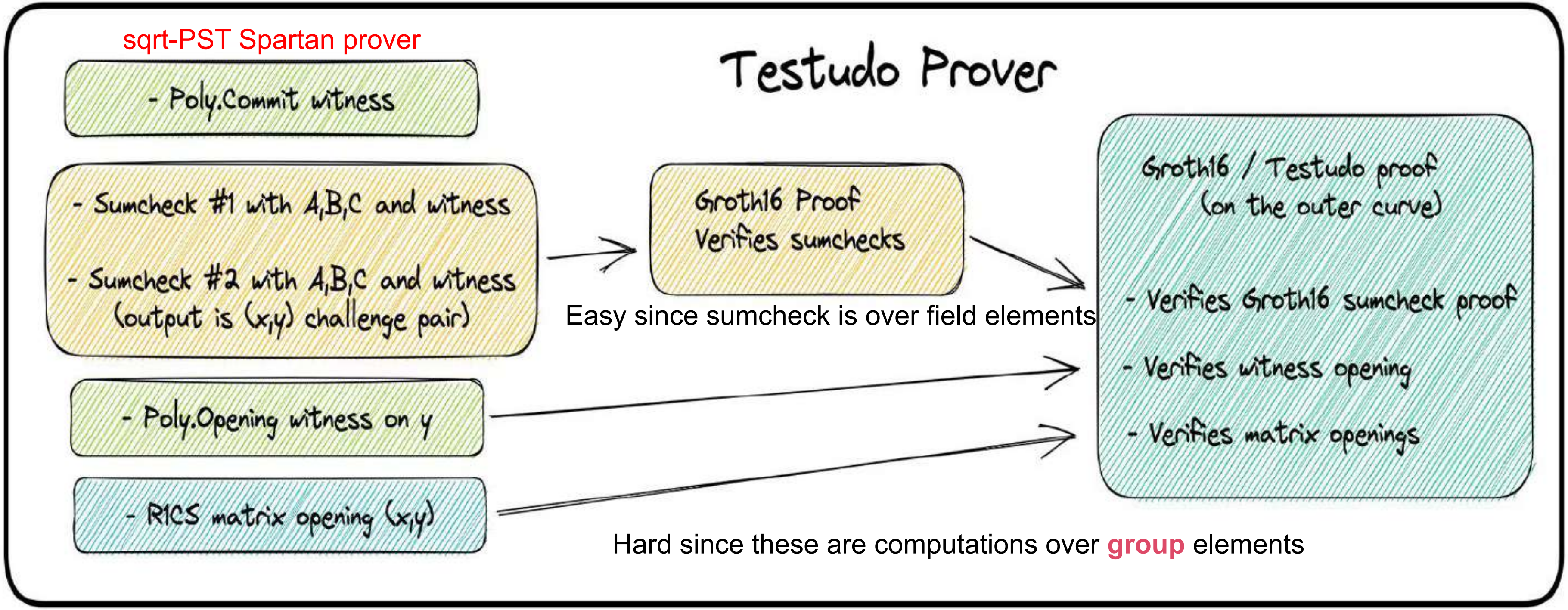


Trusted setup is O(√N) because

- polynomials now have √N terms (PST trusted setup)
- MIPP commits to √N group elements (MIPP trusted setup)
- B+'21 shows how to do this for univariate KZG commitments
  - We generalize it to multivariate PST commitments

Let’s build a SNARK with the same proof size and verification cost as Groth16, but also a *universal* and *small* trusted setup!

Testudo: PST + Spartan + Groth16

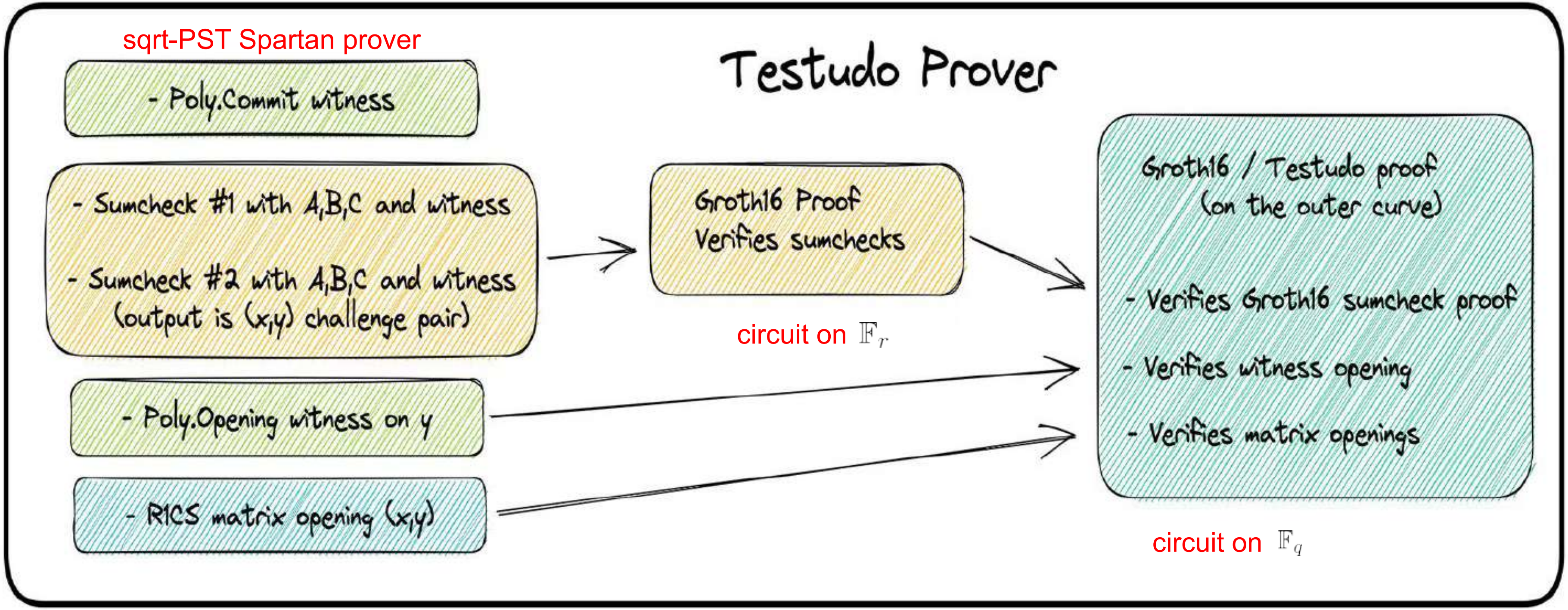


2-chain of Pairing Equipped curve

Consider one elliptic curve $E_1(\mathbb{F}_q)$ of order r (so scalar field $\mathbb{F}_r$)

- we can write a circuit with ***only scalar operations*** in $\mathbb{F}_r$ and prove on $\mathbb{F}_q$
- What if we want operations on points in the circuit?
    - if we have a second curve $E_2(\mathbb{F}_t)$ of order q, we can prove circuit with point arithmetic using E_2
- **BLS12-377** and **BW6-761** are such a pair!

Testudo: PST + Spartan + Groth16



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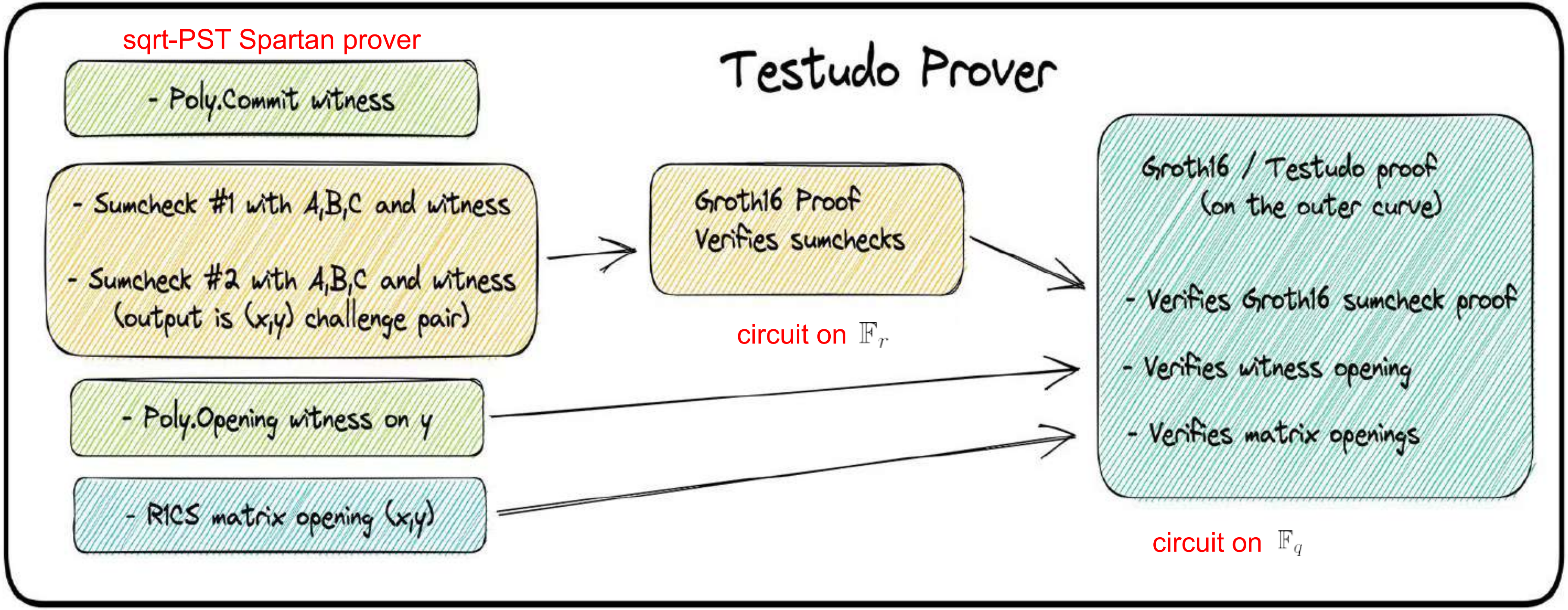

PST trusted setup + setup for ***fixed* circuit** ⇒ universal trusted setup

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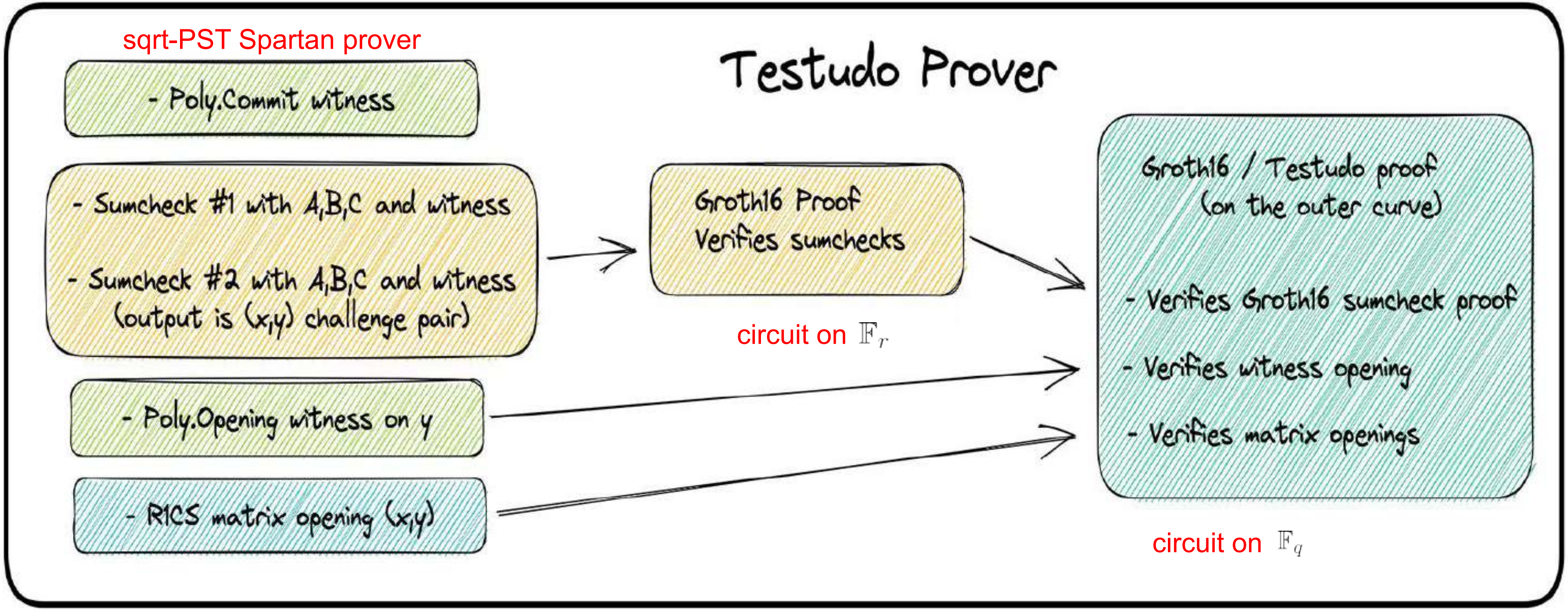
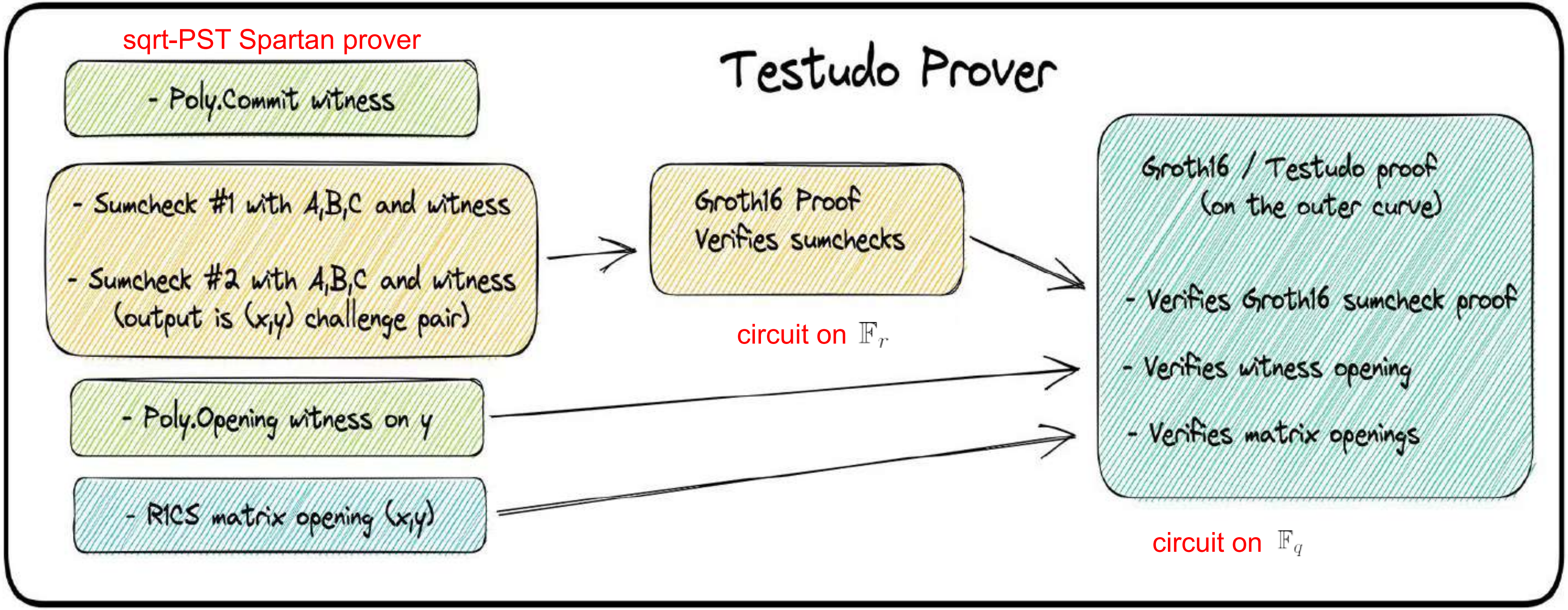


Circuit for Groth16 proof **< 10 mil constraints** => Groth16 proof generation adds **only few seconds** to proving time

Testudo: PST + Spartan + Groth16



Verifier is just one Groth16 proof verification on $\mathbb{F}_t$

Computation Commitment

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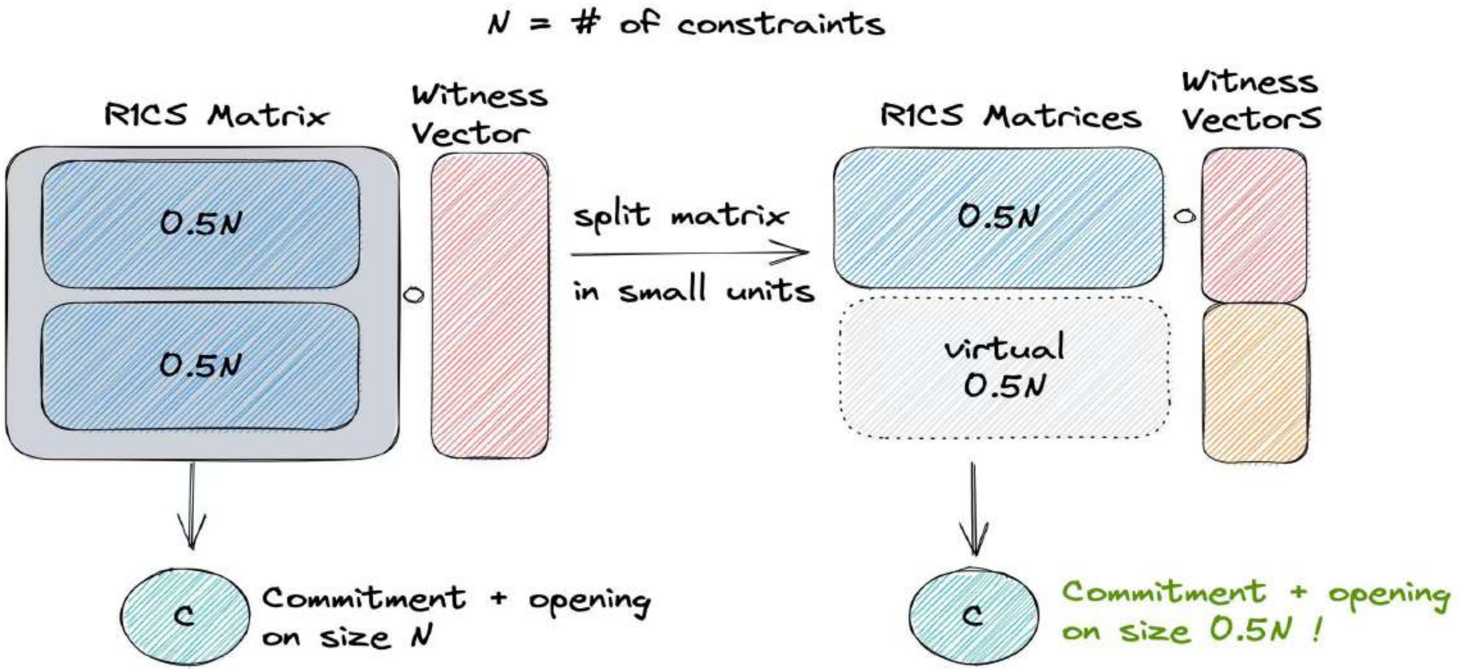

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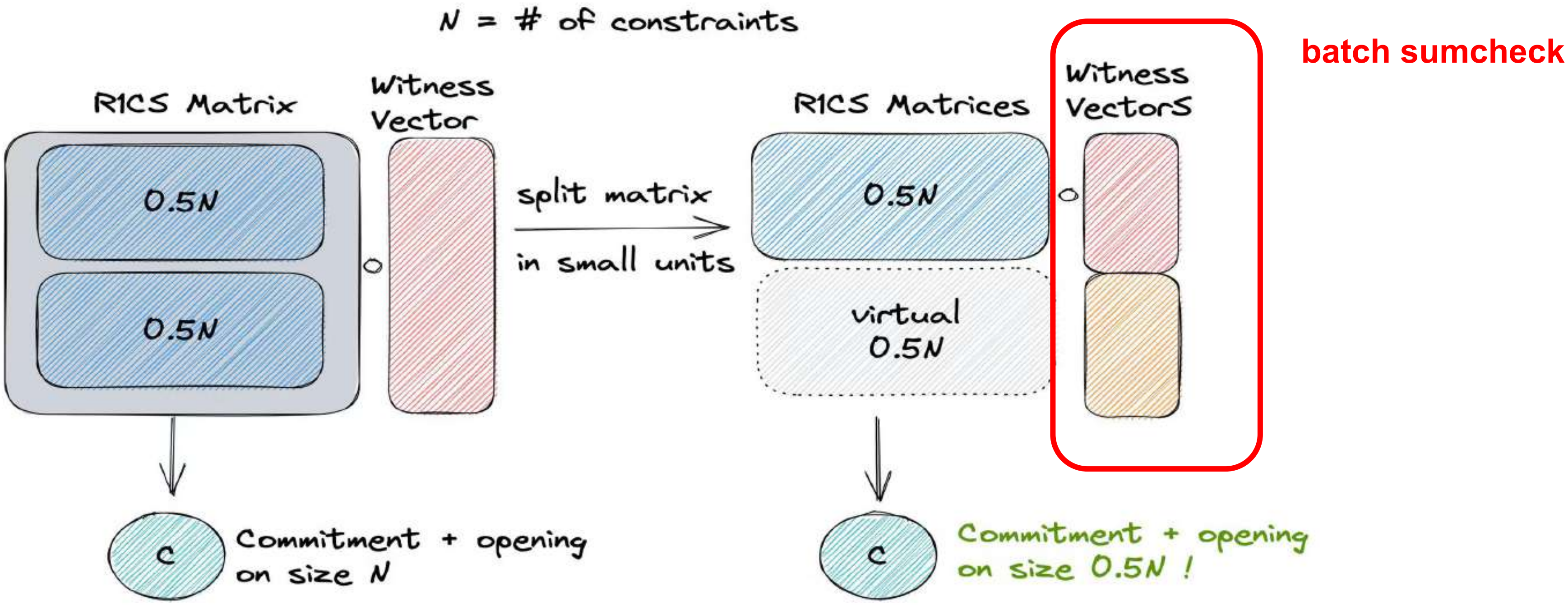

Further Steps

- Explore other ways to improve the computation commitment
  - would trusted setups help here?
- Use Dory instead of PST
  - Slower prover but much much smaller trusted setup
- Finish implementation
- Full comparison with Plonk/Hyperplonk
  - Hard to do meaningfully because of different arithmetizations
- Testudo on BLS12-381
  - more popular curve in the space, Filecoin uses this
  - lacks a “sister” curve that allows the same Groth16 compression
    - existing options do not support FFT
  - option 1: could leave the polynomial openings in the clear for proof
  - option 2: leverage another proof system for the outer circuit


Questions?