On the algebraic immunity of weightwise perfectly balanced functions

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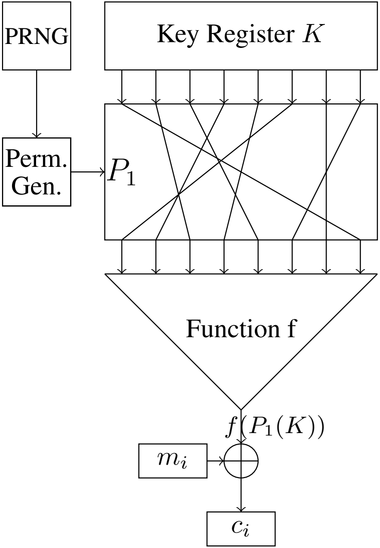
Summary

Filter Permutator and FLIP [MJSC16]

New stream cipher design adapted for homomorphic evaluation



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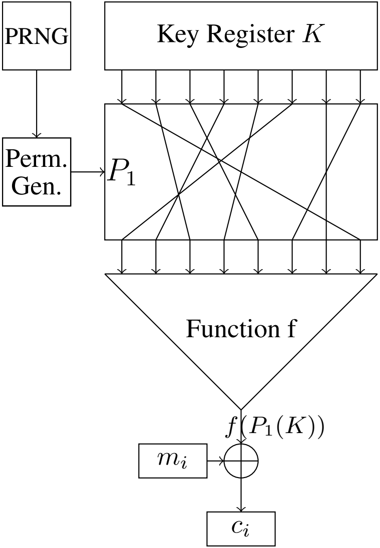

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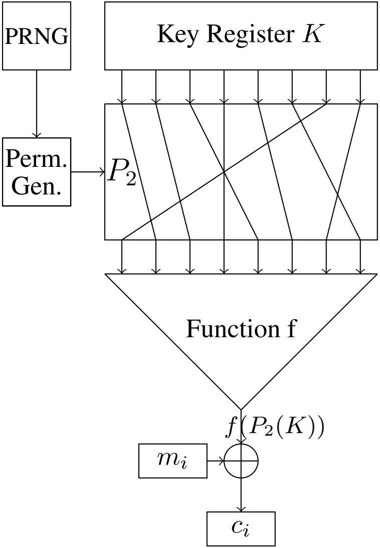

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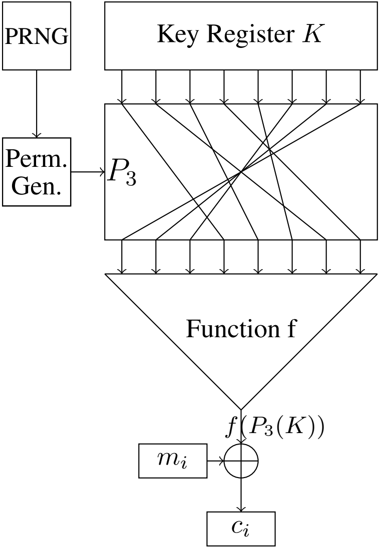

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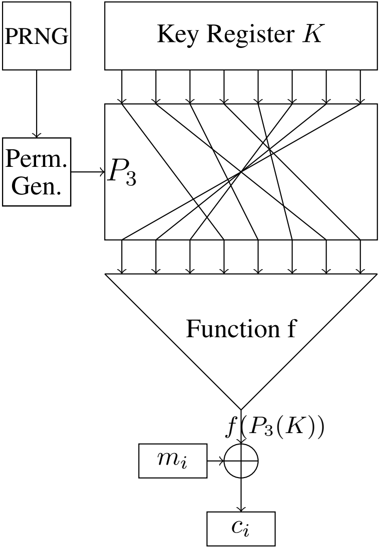

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Particularity:

$$\mathsf{w_H}(P_i(K)) = \mathsf{w_H}(P_j(K)).$$

invariant Hamming weight

Boolean functions on restricted inputs [CMR17]

Study the properties of Boolean functions applied only on a subset S of $\mathbb{F}_2^n$.

Global cryptographic criteria:

- balancedness,
- nonlinearity,
- degree,
- algebraic immunity (AI).

Restricted cryptographic criteria:

- restricted balancedness,
- restricted nonlinearity,
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Weightwise Perfectly Balanced function ($n = 2^m$)

For all $k \in [1, n-1]$:

$$|\mathsf{supp}_k(f)| = |\mathsf{E}_{k,n}|/2,$$

$f(0_n) = 0$ and $f(1_n) = 1$.
$\mathcal{WPB}_m$ denotes the set of 2^m-variable WPB functions.

WPB functions and cryptographic properties

Many constructions: CMR17, LM19, TL19, LS20, MS21, MSL21, ZS21, GM22a, GS22, MCL22, MPJDL22, ZS22, GM22b, MKL22, MSLZ22, GM23a, ZJZQ23, DM23, ...

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Algebraic Immunity

$$\mathsf{AI}(f) = \min_{g \neq 0}\{\mathsf{deg}(g) \mid f \cdot g = 0 \text{ or } (f+1) \cdot g = 0\}.$$

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Algebraic Attack, Courtois Meier 2003, adapted

- keystream bit $s_i = f(P_i(K))$,
- g such that $f \cdot g = 0$,
- $s_i = 1 \Rightarrow g(P_i(K)) = 0$, an equation of degree $\mathsf{deg}(g)$ in the key variables,
- solving an algebraic system of degree $\mathsf{AI}(f)$ gives the key.

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Former results:

- few examples in 4, 8 and 16 variables,
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Bounds on a known construction (CMR17):

$$f(x_1, x_2, \ldots, x_{2^m}) = \sum_{a=1}^{m} \sum_{i=1}^{2^{m-a}} \prod_{j=0}^{2^{a-1}-1} x_{i+j2^{m-a+1}}.$$

Proposition

$$\mathsf{AI}(f_{2^m}) \geq m, \quad \text{and for } m > 3, \quad \mathsf{AI}(f_{2^m}) \leq 2^{m-2}.$$

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$\rightarrow$ AI always at least $\mathcal{O}(\log n)$?

WPB and minimum AI (1)

Minimum degree of annihilators of WPB functions

$$\mathsf{d}_m^{\varepsilon} = \min\{\mathsf{AN}(f + \varepsilon) \mid f \in \mathcal{WPB}_m\}.$$

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Restricted Walsh transform

f a Boolean function, S a subset of $\mathbb{F}_2^n$, a an element of $\mathbb{F}_2^n$:

$$\mathcal{W}_{f,S}(a) := \sum_{x \in S} (-1)^{f(x)+ax}.$$

For $S = \mathsf{E}_{k,n}$ we denote $\mathcal{W}_{f,\mathsf{E}_{k,n}}(a)$ by $\mathcal{W}_{f,k}(a)$.

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Restricted Walsh transform to the slices

$$\mathcal{W}_{f,k}(0) := \sum_{x \in \mathsf{E}_{k,n}} (-1)^{f(x)}.$$

Lemma: $g \in \mathcal{B}_n^*$ with positive $\mathcal{W}_{g,k}(0)$ on all* slices
$\implies \exists f \in \mathcal{WPB}_m$ such that $f \cdot g = 0$ or $(f+1) \cdot g = 0$.

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Proposition: Equivalent characterization of d_m^ε

$$\mathsf{d}_m^\varepsilon = \min\{\mathsf{deg}(g), g \in \mathcal{B}_n^* \mid \forall k \in [1-\varepsilon, 2^m-\varepsilon], \mathcal{W}_{g,k}(0) \geq 0\}.$$

WPB and minimum AI (2)

First result:

$$\mathsf{d}_1^{\varepsilon} = 1 \quad \text{and} \quad \text{for } m > 1,\ \mathsf{d}_m^{\varepsilon} > 1.$$

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Corollary:

$$f \in \mathcal{WPB}_2 \Rightarrow \mathsf{AI}(f) = 2.$$

Summary

Construction from GM23a

Input: $g \in \mathcal{B}_{2^m}$.
Output: $h \in \mathcal{WPB}_m$.

Initiate the support of h to $\mathsf{supp}(g)$.
If $0_n \in \mathsf{supp}(g)$ remove 0_n from $\mathsf{supp}(h)$.
If $1_n \notin \mathsf{supp}(g)$ add 1_n to $\mathsf{supp}(h)$.
for $k \leftarrow 1$ to $n-1$ **do**
    Compute $C_{k,n} = \mathcal{W}_{g,k}(0)/2$,
    if $C_{k,n} < 0$ **then**
        remove $|C_{k,n}|$ elements from $\mathsf{supp}_k(h)$,
    else
        if $C_{k,n} > 0$ **then**
            add $C_{k,n}$ new elements to $\mathsf{supp}_k(h)$,
        end if
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Non Perfect Balancedness (NPB)

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g — small NPB, high NL $\xrightarrow{\text{Construction 1}}$ h — WPB, high NL

WPB constructions with upper bounded AI

Proposition (GM23a): h is WPB and $\mathsf{NL}(h) \geq \mathsf{NL}(g) - \mathsf{NPB}(g)$.

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Theorem

$1+g$	Construction 1 $\longrightarrow$	h
$\mathcal{W}_{g,k} \geq 0$ for $k \in [1, n]$		$\mathsf{AI}(h) \leq \mathsf{deg}(g)$

Proof intuition:

- $\mathsf{supp}(h) \subseteq \mathsf{supp}(1+g)$,
- $1+g$ annihilates g so $h \cdot g = 0$,
- g is non null.

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Examples:

◇ Porcelain functions

$\kappa_n = x_i \cdot (x_j + x_k)$

$\mathsf{NPB} = \mathsf{NL} = 2^{n-2}$ $\xrightarrow{\text{Construction 1}}$ h — $\mathsf{AI} = 2$

Cardinal (in the article): $\mathfrak{F}_8(\kappa_8) > 2^{152}$ and $\mathfrak{F}_{16}(\kappa_{16}) > 2^{44521}$.

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◇ Functions from GM23a

$\sigma_{2,n} + x_1 + \cdots + x_{n/2}$	$\xrightarrow{\text{Construction 1}}$	h
bent		$\mathsf{NL} \geq 2^{n-1} - 2^{n/2-2}$, $\mathsf{AI} = 2$

WPB constructions with lower bounded AI

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$$\begin{array}{c} g \\ \mathsf{NPB}(g) < 2^{n/2} \end{array} \xrightarrow{\text{Cons. } 1} \begin{array}{c} h \\ \mathsf{AI}(h) \geq \mathsf{AI}(g) - \lfloor \log(\mathsf{NPB}(g) + 1) \rfloor \end{array}$$

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Proposition:

$$\begin{array}{ccc} g + \sigma_{n/2,n} & & h \\ \mathsf{NPB}(g) < 2^{n/2}, \mathsf{deg}(g) < \frac{n}{2} & \xrightarrow{\text{Cons. 1}} & \mathsf{AI} \geq \frac{n}{2} - \mathsf{deg}(g) - \lfloor \log(\mathsf{NPB}(g)+1) \rfloor \end{array}$$

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Example: $d = 1 \Rightarrow \mathsf{AI}(h) \geq 2^{m-1} - m + 1$.

Summary

Conclusion and open questions

First study of the AI of WPB functions:

◇ Extreme values and distribution:

- Estimated distribution in 4, 8 and 16 variables.
- Bound on secondary constructions.
- Characterization minimum AI for all m.

◇ Constructions with bounded AI:

- bounds on GM23a's Construction.
- Upper bounded AI, many functions of AI exactly 2.
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