A Tale of VOLEs, Zero-Knowledge Proofs and Post-Quantum Signatures

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LatinCrypt 2023

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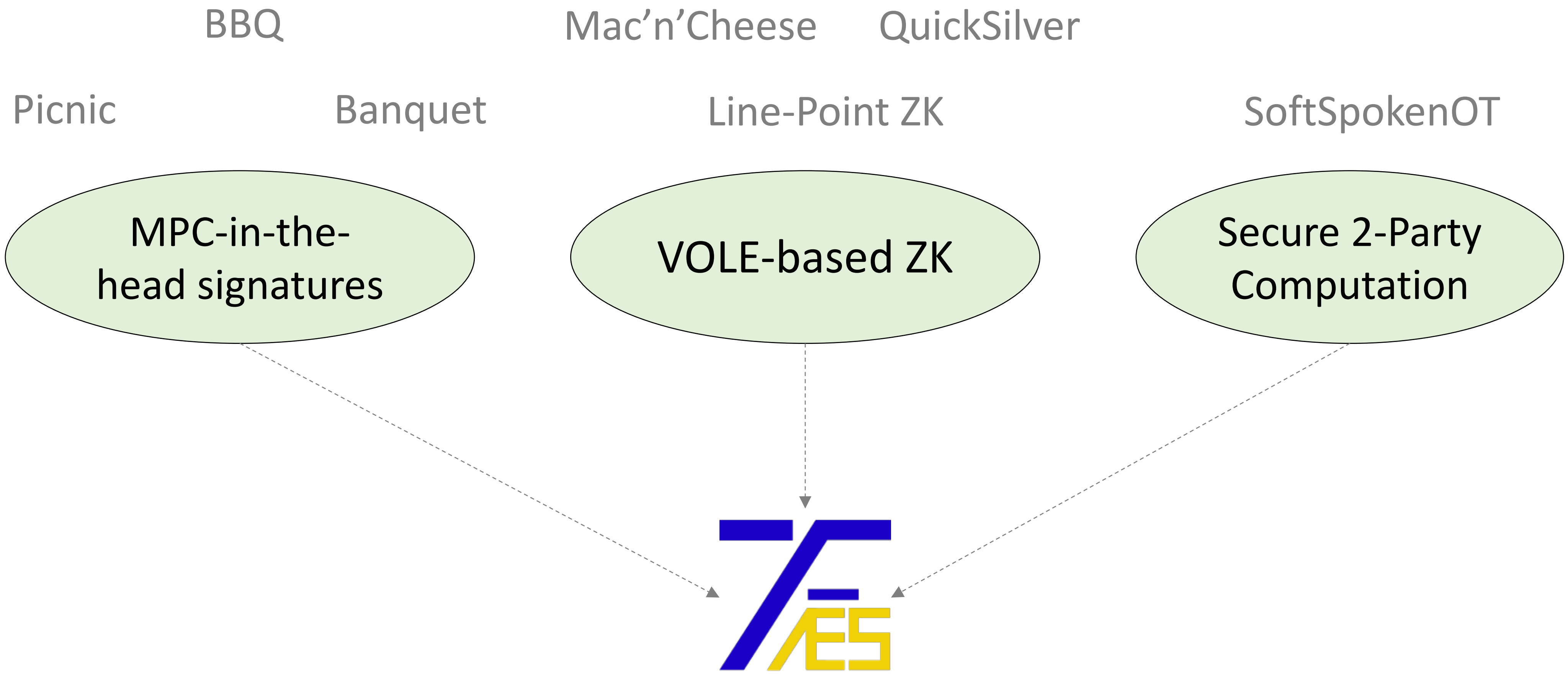
Standardization of Post-Quantum Signatures

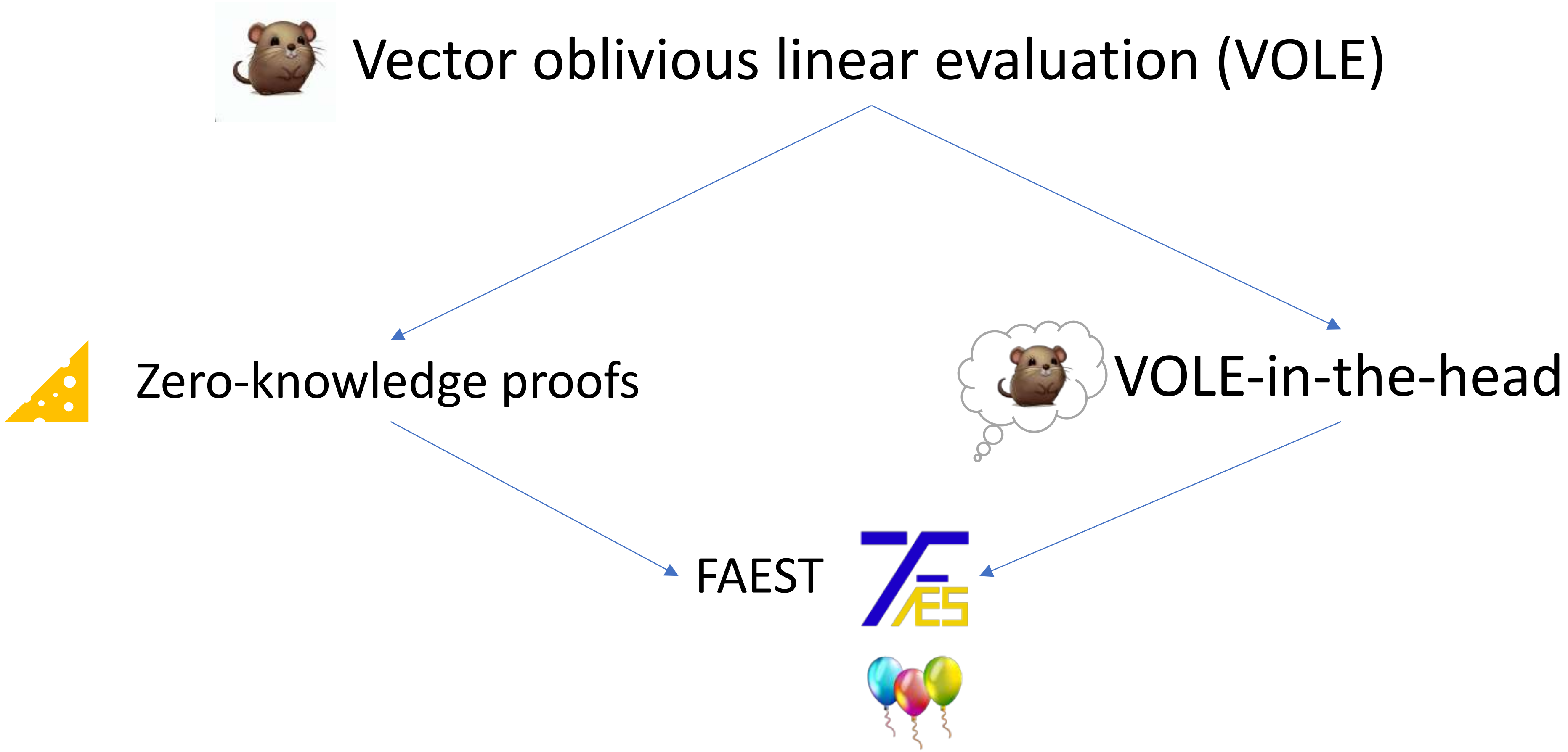
	Dilithium	Falcon	SPHINCS+	FAEST
Security:	Structured lattices	Structured lattices	Hash-based	AES/hash-based
Speed:	Fast	Fast	Slow signing	Fast-ish
Size:	2.4 kB	0.7 kB	8-17 kB	5-7 kB

2023: new algorithms submitted to diversify candidates

FAEST: Design and Inspiration



Overview of today



Based on

Publicly Verifiable Zero-Knowledge and Post-Quantum Signatures From VOLE-in-the-Head
with *Carsten Baum, Lennart Braun, Cyprien Delpech de Saint Guilhem, Michael Klooß, Emmanuela Orsini, Lawrence Roy*
CRYPTO 2023 (ePrint 2023/996)

FAEST Digital Signature Scheme
+ *Christian Majenz, Shibam Mukherjee, Sebastian Ramacher, Christian Rechberger*
Submission to NIST PQC Standardization process
https://faest.info

Zero-knowledge proofs

- A proof where the verifier learns nothing
  - Except the truth of the statement

Prover

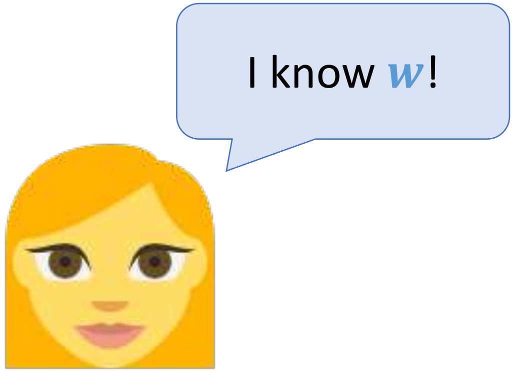
	8			3		4		
				5				1
					4	5	8	
	5	7			2		9	
9								4
	3		4			6	5	
	7	9	2					
5				6				
		6		4			2	

Verifier

Proof should be correct, sound and zero-knowledge

Zero-knowledge proofs

- A proof where the verifier learns nothing
  - Except the truth of the statement: $C(w) = 0$
  - $C : \mathbb{F}^n \to \mathbb{F}$ (arithmetic circuit)

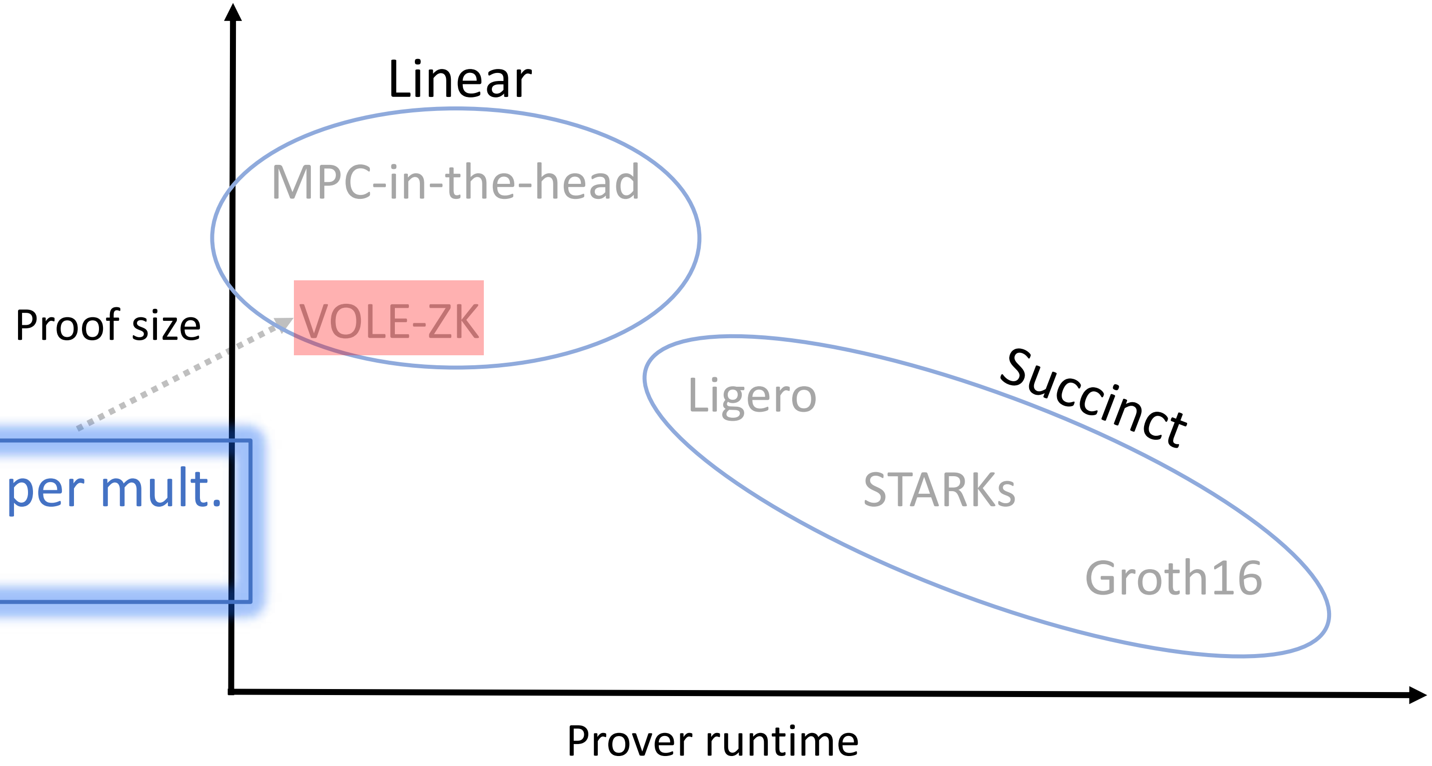


Prover

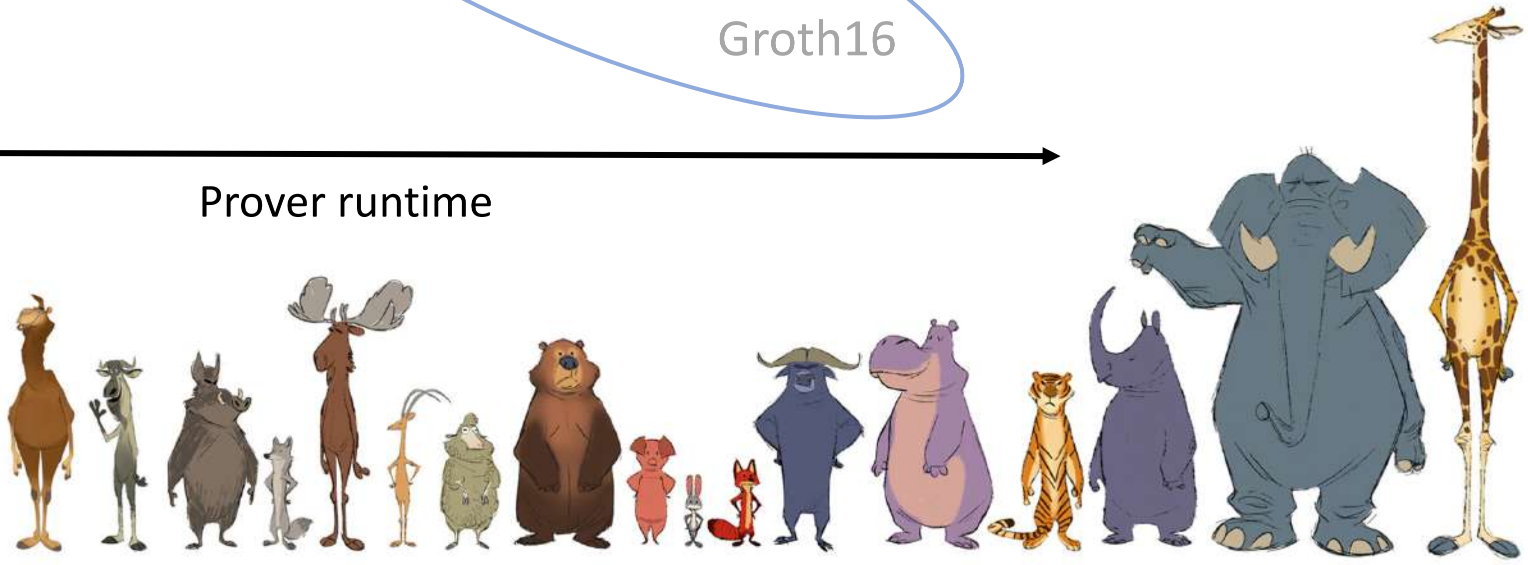
Verifier

Proof should be correct, sound and zero-knowledge

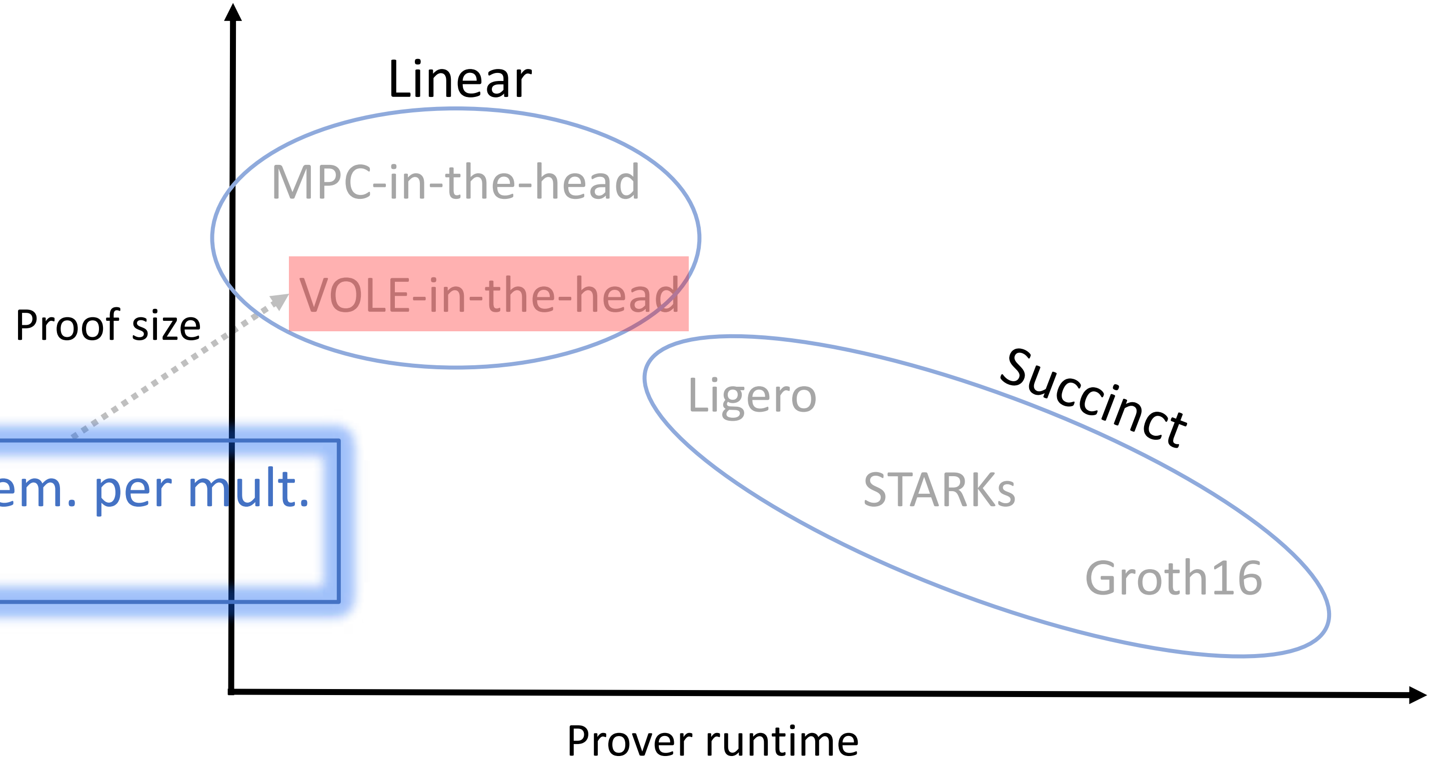
Families of ZK Proofs



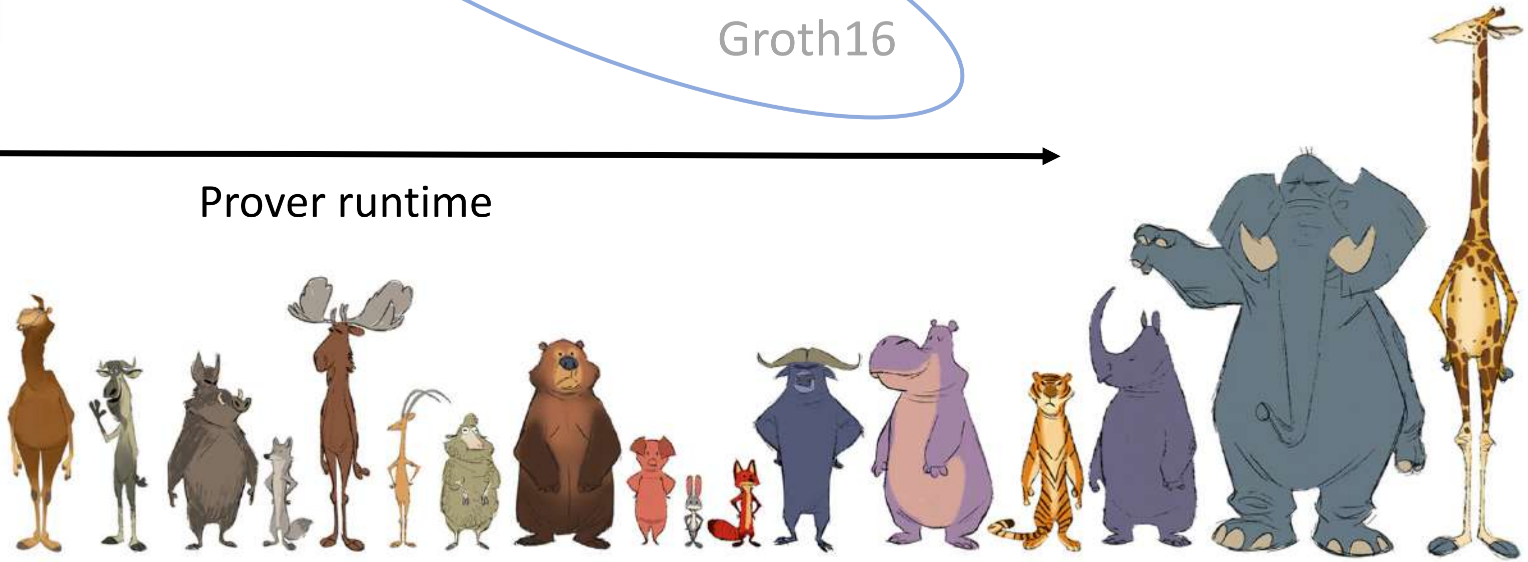
Size: < 1 field elem. per mult.
designated verifier



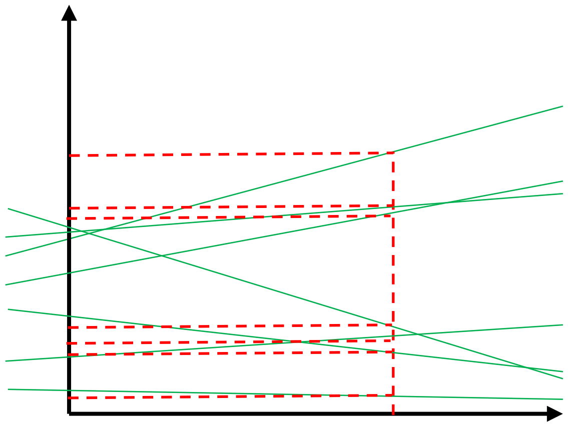
Families of ZK Proofs



Size: 1 – 10 field elem. per mult.
publicly verifiable



Vector Oblivious Linear Evaluation



$\vec{v}, \vec{w} \in \mathbb{F}^n$

VOLE

Δ

$\vec{q} = \vec{w}\Delta + \vec{v}$

$\Delta \in \mathbb{F}$

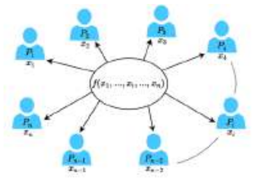
Today: $\vec{v}$ always uniform

Variant: random VOLE where $\vec{w}$ also uniform

What is VOLE good for?

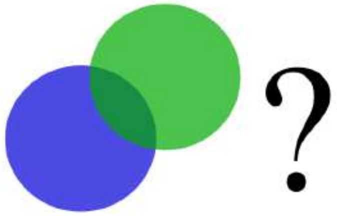
Fundamental building block in many cryptographic protocols:

- General-purpose secure computation



- Oblivious transfer
  - Implied by variant of VOLE

- Private set intersection
  - Contact discovery; online advertising

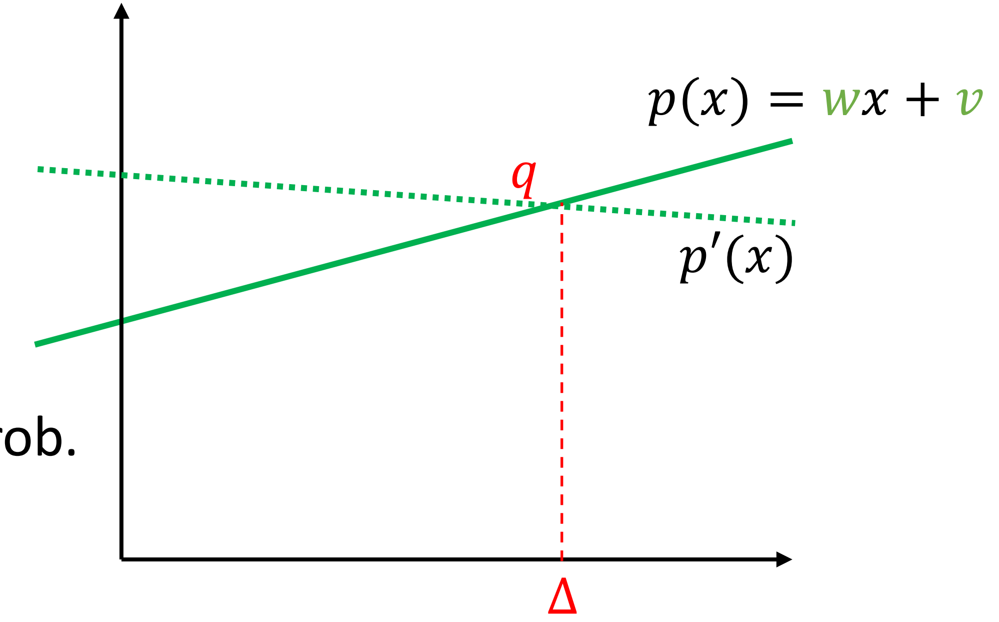


Linearly homomorphic commitments from VOLE

[BMRS 21, WYKW 21]

To commit to $\vec{w}$:

- Alice inputs $(\vec{w}, \vec{v})$ to VOLE, for random $\vec{v}$



To open w:

- Alice sends (w, v), Bob checks if $q = w\Delta + v$
- Hiding: since v is random
- Binding: opening to $w' \neq w$ requires guessing Δ, prob. $1/|\mathbb{F}|$

Commitments are linearly homomorphic

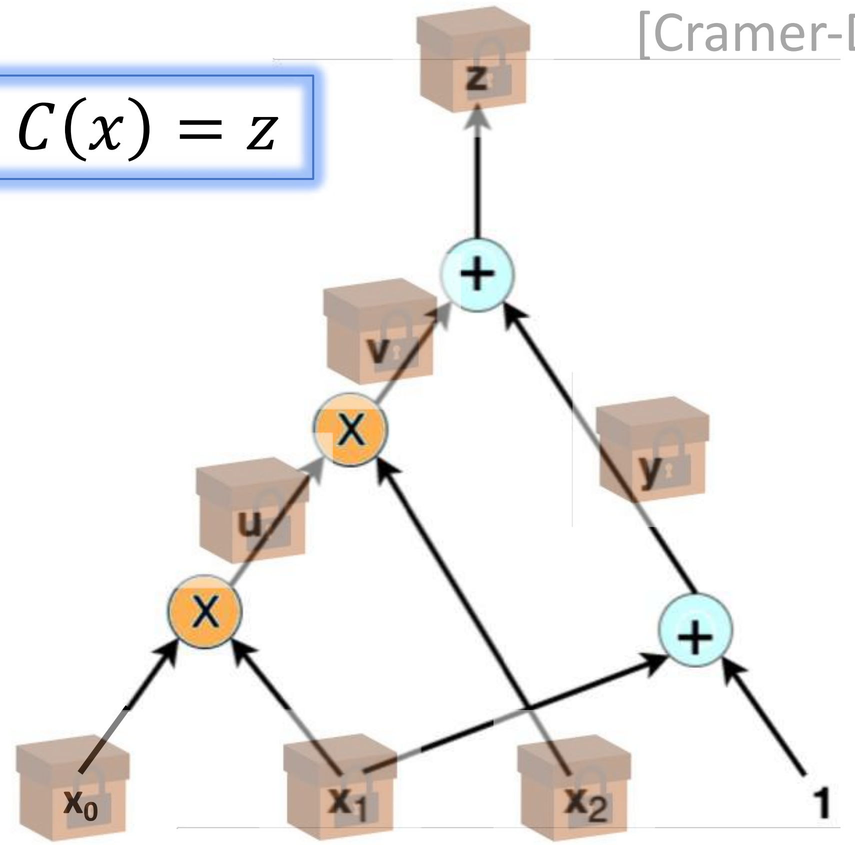
VOLE-ZK: Zero Knowledge Proofs with VOLE

Proving circuits with linear commitments

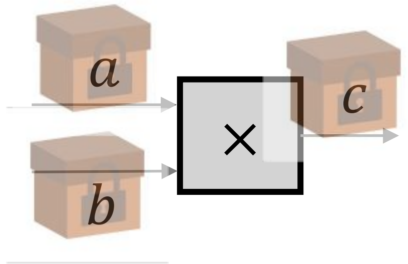
[Cramer-Damgård 97]

Goal: prove knowledge of x such that $C(x) = z$

- Commit to extended witness $\vec{w}$
  - inputs, + output wire of every mult.
- Evaluate linear gates
  - Using linear homomorphism
- Prove correctness of multiplications



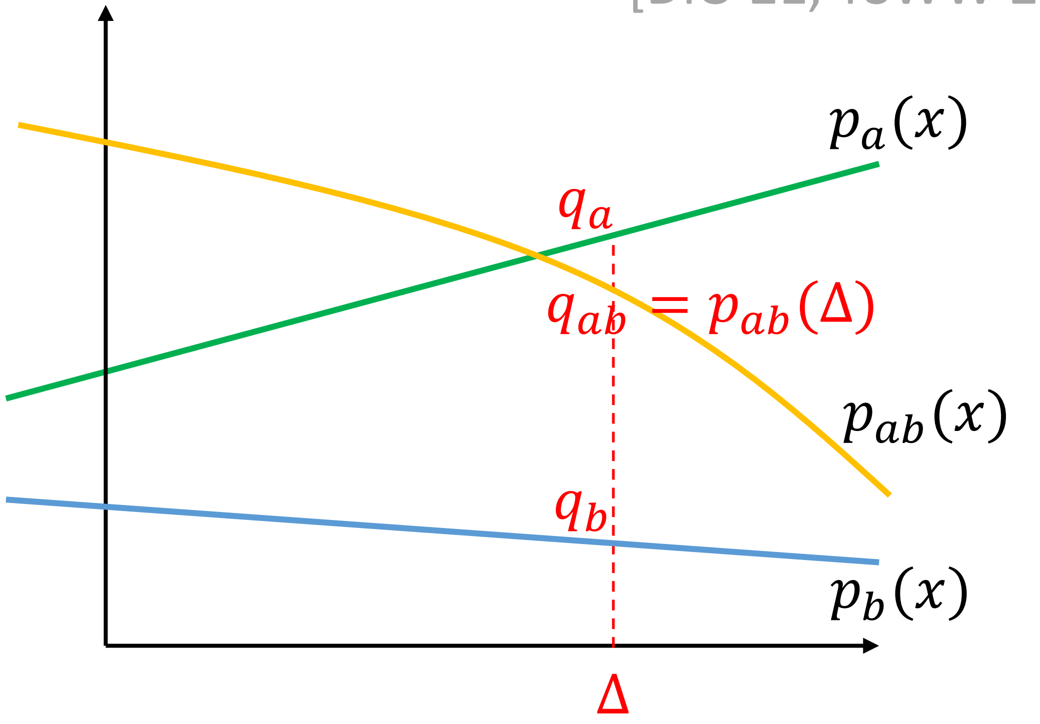
Checking multiplication gates



[DIO 21, YSWW 21]

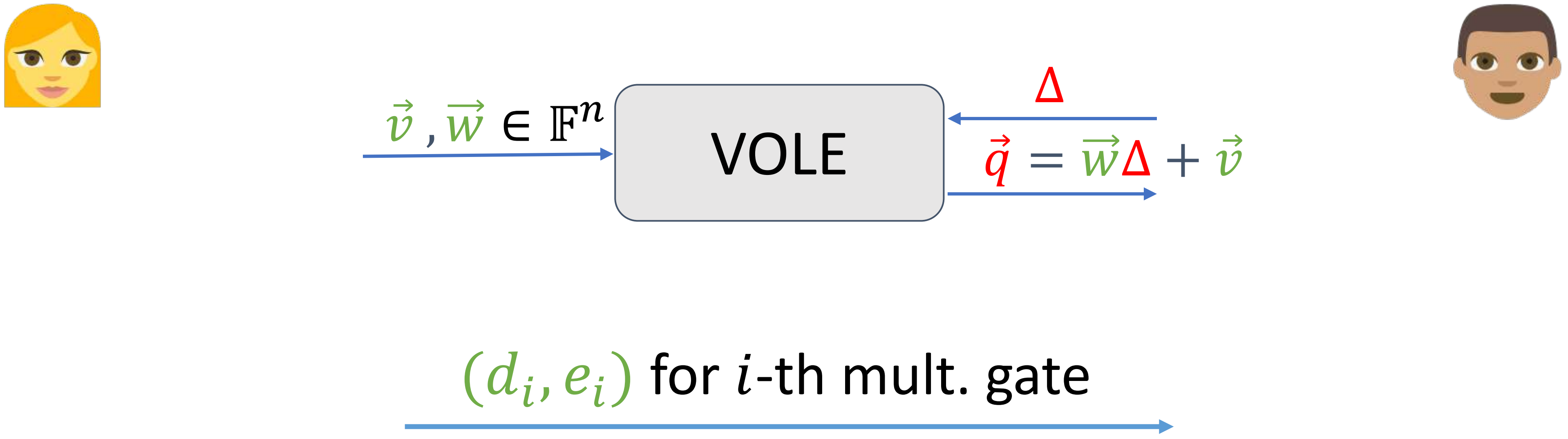
- Multiply two lines ⇒ quadratic polynomial $p_{ab}(x) = p_a(x)p_b(x)$
  $= abx^2 + \cdots$

- Compute:
  - $p_{ab}(x) - xp_c(x) = (ab - c)x^2 + dx + e$
    $= dx + e$



- Send (d, e) to Bob
  - Masked with random VOLE
  - Bob checks $d\Delta + e = q_{ab} - \Delta q_c$

ZK proof from VOLE: Initial Protocol [DIO 21]



Soundness error:

- $2/|\mathbb{F}|$

Cost for m multiplications:

- VOLE + $2m$ field elements

Optimization: batching multiplications

[YSWW 21]

$\vec{v}, \vec{w} \in \mathbb{F}^n$ → VOLE ← Δ

VOLE → $\vec{q} = \vec{w}\Delta + \vec{v}$

$r \leftarrow \mathbb{F}$

(d_i, e_i) for i-th mult. gate

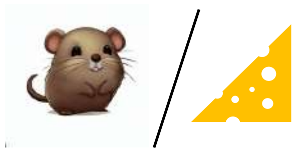
$\sum_i d_i r^i, \sum_i e_i r^i$

Soundness error:

- $2/|\mathbb{F}| + m/|\mathbb{F}|$

Cost for m multiplications:

- Length-m VOLE



Improvements/extensions

- Circuits over $\mathbb{F}_2$: [YSWW 21]
  - Let $w \in \mathbb{F}_2$, but use subfield VOLE $q = w\Delta + v$ in $\mathbb{F}_{2^k}$
- Higher-degree checks: [YSWW 21]
  - Keep adding/multiplying VOLE commitments
  - Commit to every k-th mult. gate ⇒ poly degree up to 2^k
- Mixed Boolean/arithmetic circuits [BBMRS 21, YYXKW 21]
  - VOLE in $\mathbb{F}_2$ and $\mathbb{F}_p$, prove consistency

Building VOLE

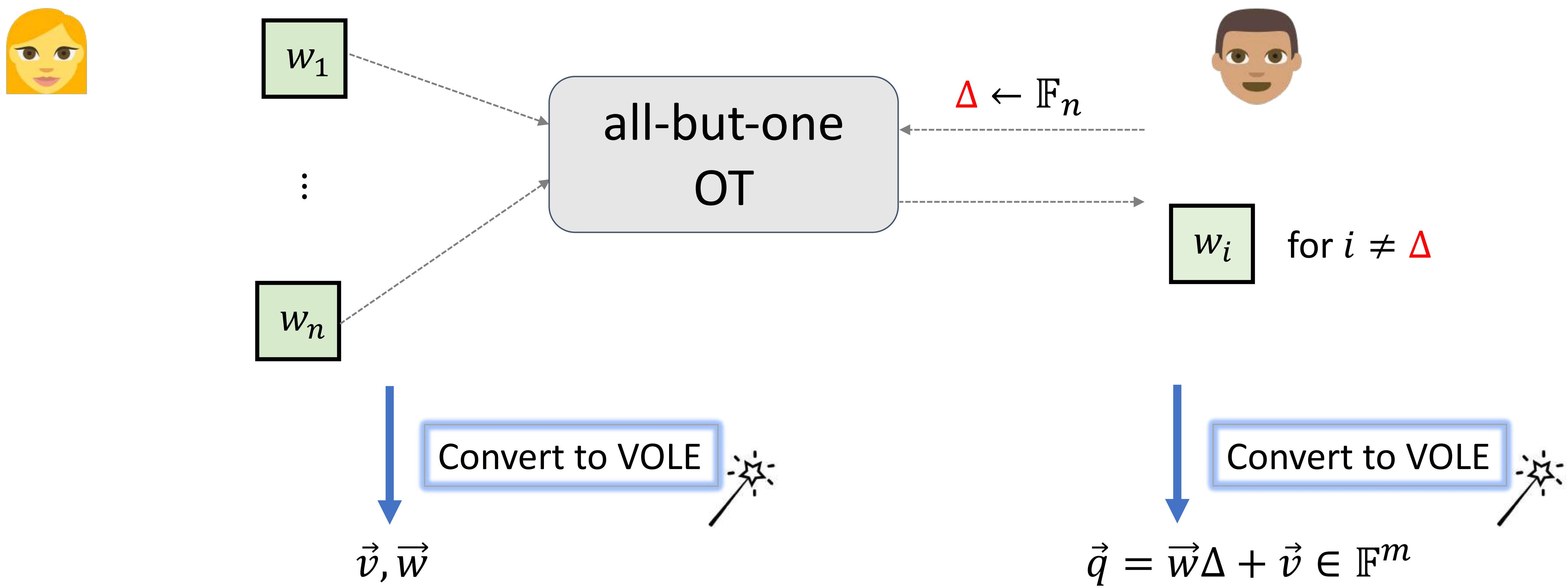
- Linearly homomorphic encryption
  - ➢ Fairly slow
  - ➢ $O(m)$ communication

- Pseudorandom correlation generators (“Silent” VOLE)
  - Learning parity with noise
  - Random, length-m VOLE: $O(\log m)$ communication ($+m$ field elem. for chosen $\vec{w}$)

- With oblivious transfer (“SoftSpokenVOLE”)
  - Mainly symmetric primitives, fast
  - $O(\log m)$ communication in small fields

Building VOLE in $\mathbb{F}_n$ with oblivious transfer (OT)

(SoftSpokenOT [Roy 22])



Conversion to VOLE

Key observation: $(n-1)$-out-of-n secret sharing ⇒ VOLE in $\mathbb{F}_n$

[Roy 22]

w_1

⋮

w_n

$w = w_1 + \cdots + w_n$

$v = -1 \cdot w_1 - \cdots - n \cdot w_n$ (in $\mathbb{F}_n$)

$\Delta \in \mathbb{F}_n$

w_i for $i \neq \Delta$

$$q = \sum_{i=1}^{n} w_i \cdot (\Delta - i)$$

$$= w\Delta + v$$

Conversion to VOLE

Key observation: $(n-1)$-out-of-n secret sharing $\Rightarrow$ VOLE in $\mathbb{F}_n$

[Roy 22]

$\boxed{\vec{w}_1} = PRG(s_1)$

$\vdots$

$\boxed{\vec{w}_n} = PRG(s_1)$

$\vec{w} = \vec{w}_1 + \cdots + \vec{w}_n$

$\vec{v} = -1 \cdot \vec{w}_1 - \cdots - n \cdot \vec{w}_n$ (in $\mathbb{F}_n^m$)

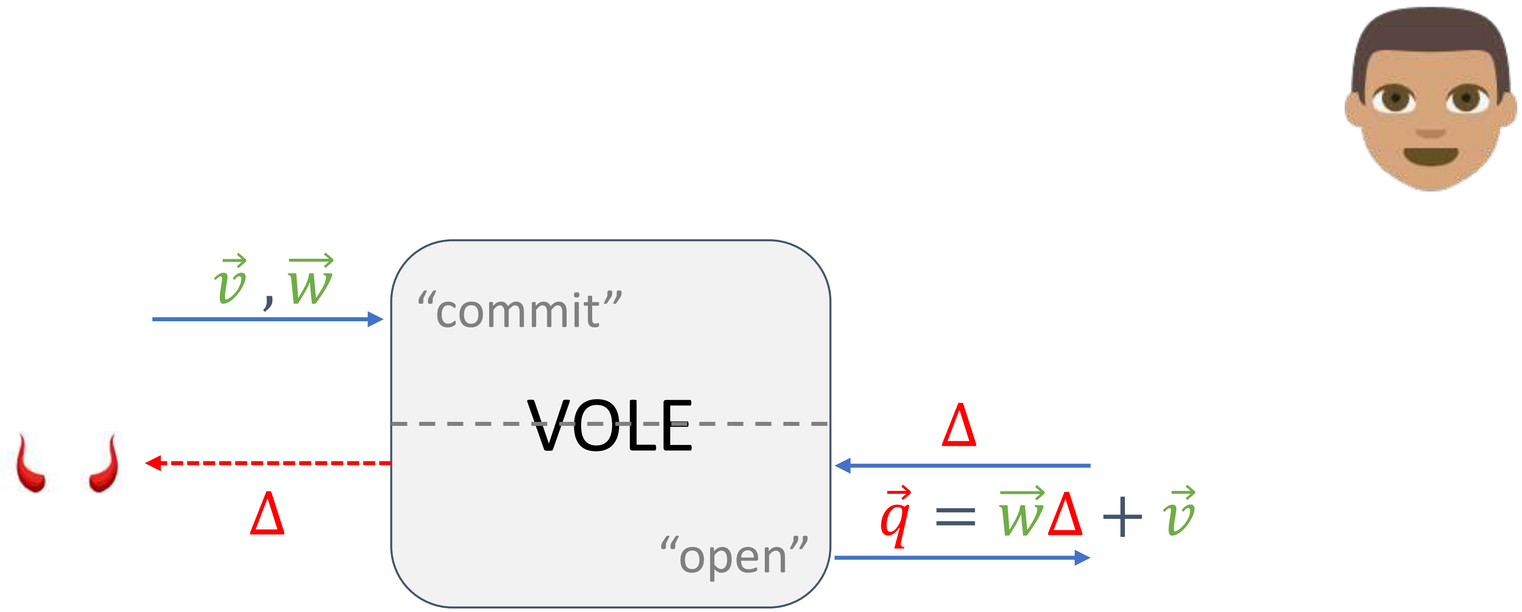
$\Delta \in \mathbb{F}_n$

$\boxed{\vec{w}_i}$ for $i \neq \Delta$

$$\vec{q} = \sum_{i=1}^{n} \vec{w}_i \cdot (\Delta - i)$$
$$= \vec{w}\Delta + \vec{v}$$

VOLE-in-the-head: from designated verifier to publicly verifiable ZK

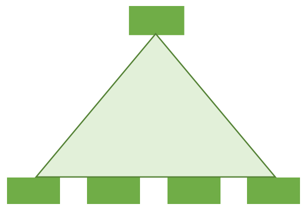
Public-Receiver VOLE (aka VOLE-in-the-head)



How to do VOLE-in-the-head? Just commit!

[BBdGKORS 23]

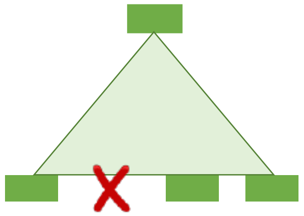
All-but-one
vector commitment



Commit to n random strings →

Challenge Δ ←

Open $n-1$ →



Convert to VOLE

$\vec{w}, \vec{v}$

Convert to VOLE

$\vec{q} = \vec{w}\Delta + \vec{v}$

VOLE-in-the-head: Summary

- If $\vec{w}$ is random, can succinctly commit to arbitrarily long VOLE
  - With PRG/hash
- Communication cost:
  - $O(\log n)$ with PRG tree optimization
- For non-random w:
  - Send extra $|w|$ field elements

ZK from VOLE-in-the-head: putting things together

VOLE: "commit" to extended witness →

$r \leftarrow \mathbb{F}_n$ ←

d, e →

$\Delta \leftarrow \mathbb{F}_n$ ←

"open" VOLE →

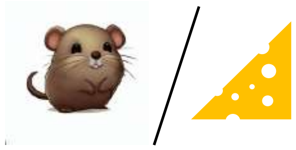
Soundness error:

- $3/|\mathbb{F}|$ (small fields)
- Improve via parallel repetition

Communication cost:

- $\mathbb{F}_2$: ≈10 bits per AND
- F_p: 1-2 field elements per mult

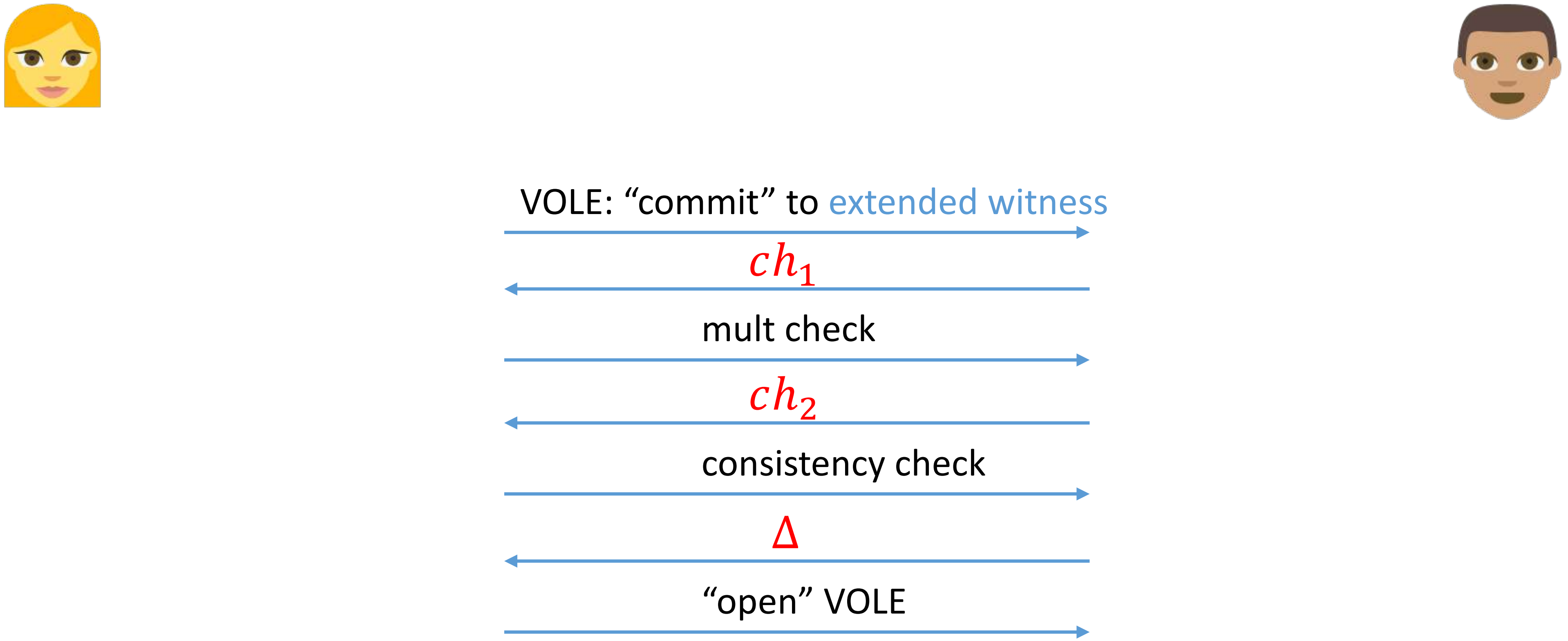
The Curse of Parallel Repetitions with >3 Rounds



- Problem: Fiat-Shamir can worsen security for >3-round protocols
  - Adversary can attack each round independently

- **Solution**: more rounds!
  - Consistency check: prove same witness is committed in small-field VOLEs
  - Allows to combine multiplication checks into one check

Final Protocol: Overview



PQ Signatures From VOLE-in-the-Head

FAEST: high-level overview

- **Public key**: AES encryption of known message under secret key

- **Signature on m**:
  - Zero-knowledge proof that key is valid
  - Using VOLE-in-the-head

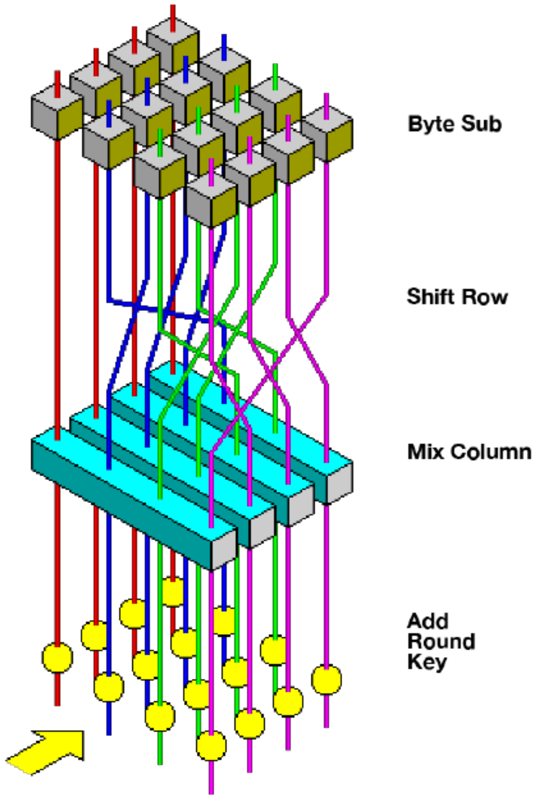
AES: a ZK-friendly block cipher?

ShiftRows, MixColumns, AddRoundKey:

- All linear over $\mathbb{F}_2$

S-Box:

- Inversion in $\mathbb{F}_{2^8}$
- Prove in ZK as 1 multiplication check



FAEST: example performance

	Sign/Verify	Size
FAEST-128s	≈ 8ms	5 006 B
FAEST-128f	≈ 1ms	6 336 B
FAEST-256s	≈ 27ms	22 100 B
FAEST-256f	≈ 3ms	28 400 B

- Signature sizes:
  - Smaller than SPHINCS+ and most code-based candidates
  - Faster signing, slower verification
- Possible variants:
  - Fixed-key AES (Even-Mansour): 10% smaller
  - MQ instead of AES: size ≈ 3 kB

Conclusion

VOLE-in-the-head ZK proofs:

- Lightweight, fast and powerful
- Proof size:
  - $\approx$ 10 bits or 1 field element per mult.

Application: FAEST PQ signature:

- Conservative security
- Reasonable performance

Resources:

- https://ia.cr/2023/996
- https://faest.info

Thank you!